MONTHLY NOTICES

OF THE

ROYAL ASTRONOMICAL SOCIETY

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MEETING OF 1931 DECEMBER 11

Dr H. Knox-Shaw, President, in the Chair.

The election by the Council of the following Fellow was duly confirmed :—

E. C. Bullard, B.A., Clare College, Cambridge.

The following candidates were proposed for election as Fellows of the Society, the names of the proposers from personal knowledge being appended :—

Clyde Fisher, Ph.D., LL.D., Curator of Astronomy, American Museum of Natural History, 77th Street and Central Park West, New York City, U.S.A. (proposed by James Stokley) ; and

Harry A. Whittaker, B.A., 48 Vegal Crescent, Ovenden, Halifax (proposed by F. A. Bellamy).

Sixty-seven presents were announced as having been received since the last meeting, including :—

E. Freundlich, Ergebnisse der Potsdamer Expedition zur Beobachtung der Sonnenfinsternis von 1929 May 9 . . . Über die Ablenkung des Lichtes in Schwerfeld der Sonne ;

G. E. Hale, Signals from the Stars.

Dr. A. J. Thompson and Mr. H. W. Newton were appointed auditors of the Treasurer's accounts for 1931.

ON A METHOD OF SOLVING KEPLER'S EQUATION.

Ernest W. Brown, Sc.D., F.R.S.

Approximate numerical solutions of Kepler's equation,

$$g = X - e \sin X,$$

have so often been given that it is hard to say whether the following is new or not. It gives high accuracy so easily, however, that a record is perhaps worth while.

Put $X = g + x$, so that $x = e \sin (g + x)$. Determine C, x_0 from

$$C \sin x_0 = e \sin g, \qquad C \cos x_0 = 1 - e \cos g,$$

or from $$C^2 = 1 + e^2 - 2e \cos g, \qquad \sin x_0 = e \sin g \div C.$$

Then the calculation of $x - x_0$ from

$$\sin (x - x_0) = -\sin^3 x_0 \div 6C \tag{1}$$

gives X with an error of order $e^5/46$ or $e^6/120$. When $e < 0{\cdot}14$ the error in X is less than $0''{\cdot}1$. Logarithms should be used for the calculation after C^2 has been found.

The proof is simple. We have

$$\begin{aligned} \sin x &= x - \tfrac{1}{6}x^3 + \tfrac{1}{120}x^5 \cdots \\ &= e \sin (x+g) - \tfrac{1}{6}e^3 \sin^3 (x+g) + \tfrac{1}{120}e^5 \sin^5 (x+g) \cdots \end{aligned}$$

Since $$\sin x - e \sin (x+g) = \sin x(1 - e \cos g) - \cos x \,.\, e \sin g,$$

the definitions of C, x_0 give

$$C \sin (x - x_0) = -\tfrac{1}{6}e^3 \sin^3 (x+g) + \tfrac{1}{120}e^5 \sin^5 (x+g) \cdots \tag{2}$$

Since C does not differ greatly from unity when e is not too great, this equation shows that the approximate solution $x = x_0$ has an error of order $e^3/6$. If we put $x = x_0$ in the first term of the right-hand member, the maximum error is found to be of order $e^6/46$. The neglect of the second term gives an error of order $e^5/120$. The next term is less than $e^7/5040$. Equation (1) is then obtained by the use of the relation $e \sin (x_0 + g) = \sin x_0$, which is easily deduced from the definitions of C, x_0.

If accuracy higher than that given by (1) is needed, it can be obtained by substituting the approximation given by (1) in the right-hand member of (2).

Well-known tables serve for stellar problems. The solution (1) is sufficiently accurate for nearly all planetary problems. It also avoids the transformation from radians to degrees. The tables of Shortrede or Bruns are convenient for finding x_0, $x - x_0$.

Yale University:
1931 *November* 11.

NOTE ON AN APPARENT INCREASE IN THE EQUATORIAL ROTATION VELOCITY OF THE SUN.

J. Evershed, F.R.S.

Spectrographic measures of the solar rotation for several years past have given values for the sidereal velocity at the equator ranging from 1·90 km./sec. to 1·94 km./sec., and values of this order have been obtained by various observers since the year 1915. The earlier measures of Dunér, 1900–03, Halm, 1904–06, Adams, 1907–08, and others up to the year 1911, all agree in giving values exceeding 2 km./sec., in close agreement with the value deduced from sunspots. The systematic observations of St. John over a series of years from 1914 show a minimum velocity of 1·90 km./sec., which remains practically constant for the ten years 1919–28, but since then the more recent values in 1929 and 1930 show a tendency to increase, the latest giving 1·95 km./sec.*

In some recent measures of the shifts of the iron and calcium lines at the Sun's limb in the region of the spectrum near H and K, I have used a device in front of the slit of the spectrograph for rotating the Sun's image through any measured angle. This is in order to compare the shifts at definite points on the Sun's limb, such as the north pole, south pole, and equator. The measures of the equatorial plates yield also the shift due to the Sun's rotation, and the values obtained during this autumn are larger than any I have observed in previous years. Like the early measures between 1900 and 1911, the mean value exceeds 2 km./sec.

As the exposure times at the Sun's limb near K are rather long with the present low altitude of the Sun, I thought it would be desirable to get other values of the rotation in the region between λ 4000 and λ 4100, where the exposure could be reduced to one minute only, and these plates so far confirm the high values shown by the H and K plates.

In all the spectra, the east and west limbs are photographed side by side near the centre of the plate, with the arc lines of iron on either side. The exposures are made successively, first on the arc, then on the two limbs, the interval between the east and west exposures not exceeding about two minutes required for readjusting the Sun's image. Lastly, a second exposure is made on the iron arc. The arc exposures are superposed near the solar spectra, but by an occulting device one end of the long arc lines has the alternate exposures separated, so that any shift occurring between the first and last exposure would be revealed. The perfect alignment of the lines in the two exposures gives the assurance that no instrumental shift can have vitiated the measurements of rotation shift. A check of this kind is very necessary, as however stable the apparatus may appear to be, very small displacements do occasionally occur, which may perhaps be ascribed to earth

* C. E. St. John, Int. Research Council, 3rd Report, *Solar and Terrestrial Relationships*, p. 121, 1931.

tremors : a movement of rotation of the end prism of the train amounting to only 1/10th of a second of arc would seriously affect the results.

All the plates have a scale of slightly over 1 mm. to the angstrom, and the total shift, west plus east, amounts to between ·05 and ·06 mm. With 15 to 20 lines measured, the probable error of a single plate is about ½ to 1 per cent. of the rotation value.

A point of some uncertainty in spectrographic measures is the correction for the distance within the limb at which the measures are made. I have arranged in this series of plates that the correction to be added will not exceed 2 per cent. of the observed value.

I give in the following table all the rotation values obtained during this autumn, with the mean value and its probable error derived from the residuals in this series.

Solar Rotation

Values of $v + v' = V$

Date 1931	Km./sec.	
July 9	2·093	
Sept. 13	2·007	
Oct. 27	1·932	
Nov. 1	2·016	15 lines, λλ 3906–3977
,, 13	1·895	
,, 21	1·954	
,, 21	2·180	
,, 29	2·101	
,, 29	2·124	
Dec. 7	1·739	20 lines, λλ 4014–4095
,, 7	2·123	
Mean	2·015 ± ·026	

In my experience of rotation measures, large variations from plate to plate sometimes occur, as appears in this series. One of the two plates obtained on December 7 yields a very low value, while the other on the same date, with all the conditions of exposure, etc., similar, gives a value in close agreement with other recent plates. It is not easy to account for this anomaly. As the stability test, when measured, shows no trace of an instrumental shift, there would seem no reason for excluding either plate from the general mean. The relative measures of arc and Sun show that the east limb shift is normal, while the west limb is largely in defect. Omitting this plate the mean would become 2·042 ± ·020.

This value happens to agree closely with the older measures of 1900–11 and with the value deduced from sunspots.

If the high values continue through the next few years, and then decrease again, a variation having a period of some thirty years would be indicated. This seems too long to be correlated with the changing polarity of sunspots, as has been suggested by St. John.*

* C. E. St. John, Int. Research Council, 3rd Report, *Solar and Terrestrial Relationships*, p. 118, 1931.

These apparent variations in the rotation affect only the gases outside the photosphere, and they amount to a change of daily angular motion from 13°·5 to 14°·8. The photosphere itself is apparently not affected by movements in the reversing layer, but has always given a constant value of about 14°·4 from the motions of spots and faculæ. The movements of the reversing layer and chromosphere seem to be greatly affected by level. Adams found that certain lines representing high levels in the reversing layer gave larger rotation values than low-level lines,* and I have been able to confirm this and extend it to the calcium lines in the chromosphere and in the prominences. In these last the daily angular rotational speed amounts to 17°,† while the place of origin of the prominence rotates with the photosphere. If the level of absorption of the lines we measure in the reversing layer has risen recently, the increase in rotation would then follow from this remarkable law of increase with height.

In my recent measures I make no attempt to differentiate between low level and high level lines, for want of sufficient material, and my results refer to the average of iron lines of mean intensity 6 in the H and K region, and 7 in the region between λ 4000 and λ 4100.

THE GALACTIC ROTATION EFFECT IN LATE TYPE STARS.

(Second Paper.)

R. O. Redman, M.A., Ph.D.

Summary.—Further radial velocity observations of seventh magnitude K stars have been made to see whether they give definite signs of the galactic rotation effect. A discussion of the errors of measurement is given, with particular reference to an unexplained apparent systematic error. Least-squares solutions are performed to find evidence for the galactic rotation effect, but the results are entirely negative. The solar motion values on the contrary are quite satisfactory.

The Observations and their Errors.—Some months ago the writer published the results of observation of the radial velocities of about two hundred K-type stars.‡ The object of these measurements was to find if any effect could be found in these velocities which could be attributed to galactic rotation. Although evidence from certain other groups of stars, particularly those of early type, has proved favourable to the theory of Lindblad and Oort, there has been less support forthcoming from late type stars, and the present work was started in an attempt to do something to fill this gap.

The stars observed were those in the Henry Draper Catalogue of type K, between visual magnitudes 7·0 and 7·5, north of −10° declination and with galactic latitude <10°. Observation has been continued, and velocities are

* W. S. Adams, *An Investigation of the Rotation Period of the Sun by Spectroscopic Methods*, p. 109 (Carnegie Inst. of Washington Pubn. No. 138, 1911).

† *M.N.*, **89**, 252, 1929.

‡ *Ibid.*, **90**, 690, 1930 ; or *Publ. D.A.O.*, **4**, 325, 1930.

now available for all these stars, rather more than 400 altogether. As before, the Cassegrain spectrograph of the 72-inch reflector at the Dominion Astrophysical Observatory, Victoria, B.C., was used with one prism and a short-focus camera. The dispersion was about 90 A/mm. at $H\gamma$. During the last few months of observation a rather lighter prism was used and the dispersion was then about 92 A/mm. Otherwise there has been no change in instrument, observer or method of working. In general only one plate was taken for each star. All the plates were measured by the writer, using the same measuring machine throughout, under uniform conditions. About 25 star lines were used on each plate.

The most important point with regard to the actual measurement of these velocities lies in the discussion of the errors, systematic and accidental. As before, observations have been made of bright standard stars, either from the I.A.U. list or from the Lick Catalogue. In 1928–29 it was found that the systematic error as given by these standard stars was $-0{\cdot}7 \pm 0{\cdot}6$ km./sec., which is negligible, particularly with these small dispersion plates. Throughout 1930, however, this quantity suffered a marked change and the systematic error, according to these standard stars, was increased to $-5{\cdot}1 \pm 0{\cdot}4$ km./sec. With the new prism, used for the last four months of observation in 1930–31, the error was $-2{\cdot}1 \pm 0{\cdot}4$ km./sec. These figures depend on 34 exposures during 1930 and 8 during 1930–31.

As an actual linear shift on the plates, this difference of about 4 km./sec. is extremely small. At $\lambda 4250$, roughly the centre of the region usually measured, 4 km./sec. = ·0007 mm., which is the wave-length of ordinary red light. With spectra of the usual dispersion of 30 A/mm., this displacement, in terms of velocity, would be less than 1·5 km./sec., which is about the size of the differences commonly found between determinations at one observatory and another.

Nevertheless, a velocity error as great as 5 km./sec. is most undesirable, and in consequence much time was spent in trying to track down the cause of the difference between 1929 and 1930. A series of careful tests were made which seemed to cover all the likely sources of error. They are summarised here ; fuller details will be found elsewhere.*

An extra diffusing screen had been put in the optical train for the comparison arc, in order to make the exposures more controllable. (The reason for this was that faster plates had come into use.) Tests, with and without the screen, revealed that this had no effect on the velocities. The collimation of telescope with spectrograph was checked and corrected ; this made not the slightest difference to the errors. Earlier plates were remeasured, to test for new personal errors of measurement, with negative result. Some of the programme stars were repeated (normally only one photograph was taken for each) to see whether the changing error might not be confined to the standard stars. The change was clearly present in the actual programme stars. Finally, guiding tests were made to see whether some alteration in the method of following the star had been made unconsciously by the observer. These tests showed that it would be difficult for deliberately very bad guiding

* *Publ. D.A.O.*, **6**, No. 5 (in press).

to give rise to an error as great as this ; in the normal course of events any error from this source should be very small.

At the time no other observer was using the same short-focus camera for radial velocity determinations ; with the longer cameras used by others no systematic change was detected during this period. This supports the observation that, as far as is known, no change had taken place in the instrument itself.

This seemed to exhaust the possible sources of this trouble, and the writer was obliged to leave the matter at this rather unsettled stage. The only satisfactory feature is that as an actual measured quantity, *i.e.* as a displacement on the photographic plate itself, the error is quite small.

It happens that this particular error makes little difference to our conclusions concerning the " galactic rotation " effect in this group of stars. It is nevertheless recommended that the following corrections be applied to the velocities :—

1928–29 *	+1 km./sec.
1930 †	+5 ,,
1930–31 (new prism) †	+2 ,,

The accidental error of the velocities was estimated in the earlier account as 3·5 km./sec. (probable error). There are now a greater number of stars with more than one measurement each, 38 all told, and from the interagreement of these the probable error per plate was found to be 3·6 km./sec., using velocities corrected as above.

Only three stars in the whole list have had radial velocities determined at other observatories, and two of these three have definitely peculiar spectra, so that virtually no information may be obtained from this source.

As far as our evidence goes, we can still regard the probable error of the corrected velocities as being between 3·5 and 4 km./sec. per plate.

The experiences related above led to some consideration of the methods of radial velocity determination as ordinarily carried out to-day. In the past a good deal of such work has been concerned with comparatively bright stars, and each observatory has had a fair number of velocity determinations in common with other institutions, so that the intercomparison of results has made it possible for systematic errors to be evaluated. Now, however, observations are being directed more and more to fainter stars, and spectrographs of very small dispersion are likely to be used to an increasing extent. It will be uneconomical to allow much overlapping of work on these more difficult objects, and thus it becomes the more necessary that test plates of some kind be taken regularly and systematically in order to check the performance of both instrument and observer.

The necessity for such precautions has been recognised for many years at the Lick Observatory, but does not seem to have been paid sufficient attention elsewhere. As test objects the Lick observers originally used the planets whose radial velocities with respect to the earth are known accurately from

* *Publ. D.A.O.*, **4**, No. 20, Table 5, 1930.
† *Ibid.*, **6**, No. 5, Table 9 (in press).

independent considerations. It is, however, much better to use actual stars, since with planets there is little question of guiding error or irregular illumination of the slit. Further, the standard stars, if possible, should be of the same type and about the same apparent magnitude as those whose velocities are being investigated. The writer understands that this last condition is observed at the Lick Observatory, where stars are now being used. It is most desirable that such a method should be made the standard procedure in all radial velocity work, wherever carried out.

The list of standard stars provided by the I.A.U.* is most valuable in this respect, but it needs extending to fainter magnitudes. In addition a closer scrutiny might be given to stars already included. α Lyræ has recently been reported as a variable,† and some observers have questioned the constancy of even α Boötis.

The Galactic Rotation Effect.—This series of radial velocity observations was undertaken primarily in order to investigate whether definite signs of galactic rotation could be found in late type stars, as in those of types O and B. The procedure in calculating the solar motion and galactic rotation elements is the same as that followed before, which in turn is similar to that used by J. S. Plaskett.‡

The complete list contained 425 stars, 15 at the galactic pole and most of the remaining 410 within 10° of the galactic equator. A certain number of these were omitted from the calculations, the criteria for rejection being : radial velocity > 150 km./sec., or total annual proper motion > 0″·15, or a dwarf or " dwarfish " spectrum, or a definitely peculiar spectrum. The rejections for high velocity are deliberately rather conservative. We know no certain clear-cut dynamical distinction between moderately high velocity stars and slower objects of the same spectral characteristics, so that the drastic rejection policy followed by some workers has no clear justification for it. After the rejections mentioned, 392 stars remained, 380 in or near the galactic plane and 12 near the galactic pole. They were divided into 23 groups and a least-squares solution performed, using an equation of condition of the form

$$V_0 \cos D + \bar{r}A \sin 2(l - l_0) \cos^2 b = \text{observed velocity}.$$

V_0 is the solar velocity relative to this group of stars, expressed in km./sec., D is the angular distance of the solar apex from the centre of each group, $\bar{r}$ is the average distance of these stars from the Sun, A is a constant, (l, b) are the galactic co-ordinates of the centre of each group and l_0 is the longitude of the galactic centre. In order that the least-squares method may be applied, the equation is put in the form

$$(V_0 \cos B \cos L) \cos b \cos l + (V_0 \cos B \sin L) \cos b \sin l + (V_0 \sin B) \sin b + (\bar{r}A \cos 2l_0) \sin 2l \cos^2 b - (\bar{r}A \sin 2l_0) \cos 2l \cos^2 b = \text{observed velocity},$$

where (L, B) are the galactic co-ordinates of the solar apex.

* *Transactions of the International Astronomical Union*, **3**, 171, 1928.

† Guthnick, *Strömungen in Sternatmosphären*, Erste Mitteilung, α Lyræ (Berlin 1931).

‡ *M.N.*, **88**, 395, 1928.

In the evaluation of solar motion, etc., it is the general practice to include a K term, that is a constant term relating either to an apparent general expansion or contraction of the stellar system, or to a systematic error in the velocities. Such a course is useless here. Owing to the fact that the stars are distributed over little more than half the sky it is practically impossible to distinguish between a K term and a component of the solar motion. The least-squares method will in general give a finite value for the K term, if it is included, but the figure will have little or no significance. We therefore omit it entirely.

The results of the least-squares solution, with probable errors of the calculated quantities included, are—

$$\begin{aligned} V_0 &= 21{\cdot}5 \pm 1{\cdot}4 \text{ km./sec.} \\ L &= 32^\circ \pm 3^\circ \\ B &= +16^\circ \pm 8^\circ \\ \bar{r}A &= 1{\cdot}9 \pm 1{\cdot}3 \text{ km./sec.} \\ l_0 &= 17^\circ \pm 18^\circ. \end{aligned}$$

The sum of the squares of the residuals is 509·8.

A second calculation was performed neglecting the galactic rotation terms and considering only solar motion. The results are—

$$\begin{aligned} V_0 &= 20{\cdot}5 \pm 1{\cdot}3 \text{ km./sec.} \\ L &= 31^\circ \pm 3^\circ \\ B &= +17^\circ \pm 9^\circ. \end{aligned}$$

The sum of the squares of the residuals here is 532·2.

The probable error of an observational equation of unit weight is in each case 3·2 km./sec. It is clear that little improvement has been made in the representation of the velocities by including the second harmonic terms, for the sum of the squares of the residuals has been reduced merely by 4 per cent.

The solar motion is practically the same in each solution. The apex $L=32^\circ$, $B=16^\circ$ is in equatorial co-ordinates $\alpha=280^\circ$, $\delta=+35^\circ$. This is not so far north as the position determined in the previous paper, but it is still to the north and east of the apex for the naked-eye stars, which is about $\alpha=270^\circ$, $\delta=30^\circ$. The following is a brief list of other determinations of the position of the apex for the stars of spectral type similar to ours :

α	δ	Source
°	°	
276	35	Wilson and Raymond, *A.J.*, **40**, 121, 1930.
279	43	Schilt and Miss Barney, *A.J.*, **40**, 175, 1930.
272	36	,, ,, ,, *A.J.*, **37**, 187, 1927.
287	54	Schilt, *A.J.*, **38**, 34, 1927.
282	43	Strömberg, *Mt. Wilson Cont.*, **245**, 300, 1922.

The mean of these five positions is $\alpha=279^\circ$, $\delta=42^\circ$, *i.e.* $L=38^\circ$, $B=18^\circ$. The agreement with our values is quite as good as the interagreement of these other determinations, and it is satisfactory to notice how near the

figures just calculated are to those given by Wilson and Raymond, which are probably the most reliable of the five quoted. It should be remembered that owing to the distribution of the stars in the sky, the latitude of our apex B is subject to considerable uncertainty.

Our solar velocity is somewhat higher than the 17·9 deduced by Wilson and Raymond or 18·2 by Strömberg, but any value close to 20 km./sec. cannot be regarded as unsatisfactory for such a group of stars.

By combination of radial velocity and proper-motion data a mean parallax may be found. In the earlier paper the result was 0″·0047. Repeating the computation with the additional velocities now available and the solar motion just obtained, a revised value, 0″·0053, was found. Since individual parallaxes will be scattered about this mean value, the average distance will not be the reciprocal of the mean parallax, but will be perhaps 20 per cent. greater,* so that $\bar{r}$ should be about 230 parsecs. Thus, since A is approximately 0·017 km./sec. parsec,† $\bar{r}A$ should be about 4 km./sec. Our value is $1{\cdot}9 \pm 1{\cdot}3$.

The l_0 found should be the longitude of the galactic centre, which astronomers agree in putting near 325° or 330°. Our value is $17° \pm 18°$, no less than 50° away. The l_0 derived in the previous paper showed a similar discrepancy. The probable error is an admittedly big quantity, but it is too small to account for a difference as great as 50°. In addition, it must be remembered that the actual term considered is a second harmonic; actually the discrepancy in phase is 100°.

In brief, the value $\bar{r}A$ given by these stars differs from the expected value by more than one and a half times the probable error, the calculated l_0 differs from its expected value by more than two and a half times the corresponding probable error, and finally the velocities are represented practically as well by the solar motion alone as when second harmonic terms are included. It is legitimate to conclude that little sign of a " galactic rotation " effect is shown by these stars.

This result cannot be explained as due to the application of erroneous corrections to the velocities. If we take the latter precisely as observed and carry through the calculation again we get

$$\begin{aligned} V_0 &= \ \ 24{\cdot}8 \pm 1{\cdot}4 \text{ km./sec.} \\ L &= \ \ 42° \pm 3° \\ B &= +18 \ \pm 8°{\cdot}5 \\ \bar{r}A &= \ \ 3{\cdot}9 \pm 1{\cdot}3 \text{ km./sec.} \\ l_0 &= \ \ 37° \pm 18°. \end{aligned}$$

Although $\bar{r}A$ is now of the expected size, l_0 is farther off than ever from the recognised galactic centre. The chief change made by the corrections appears in the solar motion.

An inspection of other results from stars within 350 parsecs of the Sun (Table I) shows that in general the agreement between theory and observation is not at all close. For more distant O and B stars and the interstellar

* Oort, *B.A.N.*, **132**, 82, 1927.
† J. S. Plaskett, *M.N.*, **90**, 626, 1930.

calcium the results, which have been given by Plaskett and Pearce,* are more satisfactory, and the value of $\bar{r}A$ for the calcium being half that for the stars in whose spectra the " detached " lines are found is a strong support for the theory. It should be borne in mind, however, that these last results are derived from stars in the northern sky only.

TABLE I

Galactic Rotation Results from Radial Velocities of Stars within 350 Parsecs of the Sun

	Type	Apparent Magnitude	Approx. Distance	No.	$\bar{r}A$	l_0	Source
						°	
(1)	O–B2	3·98	250	78	2·8	290	Plaskett and Pearce.
(2)	B3–B5	4·64	135	241	1·0	300	,, ,,
(3)	B3–B5	6·60	330	222	3·2	346	,, ,,
(4)	A–G ($\mu < ''·020$)	5·3	180	122	7	345	Oort, *B.A.N.*, **132**, 80, 1927.
(5)	A–G ($\mu < ''·020$)	> 6·4	210	88	10	337	,, ,,
(6)	B8–B9	5·1	160	250	2·1	334	Lindblad, *M.N.*, **90**, 580, 1930.
(7)	F–K ($\mu < ''·040$)	5·4	190	714	3·8	346	,, ,,
(8)	A ($\mu < ''·040$)	5·3	160	304	5·0	319	,, ,,
(9)	K	7·3	230	392	1·9	17	This paper.

(1) (2) (3) These and other results for the O to B5 stars were reported in the George Darwin Lecture for 1930 (J. S. Plaskett, *M.N.*, **90**, 633, 1930). The negative $\bar{r}A$ there for the second group is a misprint.

(4) (5) Oort's other groups have been omitted since they have been replaced by later determinations or correspond to stars at greater distances than 300–400 parsecs.

(7) (8) (9) The data are mostly the same as Oort's.

Proper motions in which there should also be a second harmonic term due to " galactic rotation," have been used by various workers to give values of l_0, the longitude of the galactic centre. The principal results are in Table II. It may be noted that most of the values for l_0 are greater than 330°, a tendency to be seen also in the radial velocity results. It may be mentioned that Dyson in his investigation found a third harmonic with a coefficient larger than that of the second.

Oort has shown † that, provided the galaxy is in a dynamically steady state, and the system is symmetrical both with respect to the galactic plane and about the axis of rotation, and that the velocities are distributed according to an ellipsoidal law, then the major axis of the ellipsoid should be directed towards the galactic centre. It is a well-known fact of observation that the direction of this axis, although it may vary considerably according to the particular group of stars considered, is generally in the neighbourhood of

* J. S. Plaskett, George Darwin Lecture, *M.N.*, **90**, 633, 1930.

† *B.A.N.*, **159**, 274, 1928.

galactic longitude 345° or 350°. (Our particular group of K stars gave 339°.)

Table II

Values for l_0 from Galactic Rotation Effect in Proper Motions

Stars	l_0	Source
	°	
O, B, c and N types	326	Oort, *B.A.N.*, **132**, 86, 1927.
Boss stars	333	,, ,, ,,
,,	351	Schilt, *A.J.*, **29**, 20, 1928.
Gyllenberg Zone Catalogue	340	,, ,, ,,
Prager ,, ,,	337	,, ,, ,,
Yale 50°–55° ,,	{348 / 360}	Merrill, *A.J.*, **29**, 90, 1929.
Boss stars (Bo–Ao)	330	Dyson, *M.N.*, **90**, 233, 1929.

Both the proper-motion data and the figures for stream motion refer to a considerable extent to stars fairly near the Sun. The evidence provided by these and the radial velocities of the nearer stars is certainly not too favourable to the rotation theory. In so far as these objects support the theory, they tend to behave as though the galactic centre were at a greater longitude than the usually accepted value.

It is possible that local irregularities are to be held responsible for swamping the general effect of the galactic rotation for the nearer stars (where the effect is in any case small). We can see by a mere glance at the sky that the galactic system is not an ideal homogeneous flattened disc, and we know that there exist star clouds and clusters, at least some of which have group motions of their own. Indeed, Shapley * has suggested that the Milky Way system is a super-galaxy, a cloud of some hundreds of galaxies, or spiral nebulæ, similar to groups observed in the Coma-Virgo or Centaurus regions. The usual features of stellar motions (except high velocities) are then to be regarded as merely local phenomena.

Although the picture is an attractive one in some respects, it would, if accepted, compel us to abandon the simple hypotheses of Lindblad and Oort, which have provided such satisfactory explanation of some of the prominent features of stellar movements.

A local cluster rotation has been proposed by Mineur. It will provide not only a second harmonic term in stellar radial velocities or proper motions, but also first, third, and other harmonics. The first will combine with the solar motion and may provide a reason why different groups of stars give slightly different values for this quantity. The third harmonic has been observed in proper motions, but here there can be difficulty with systematic errors. Radial velocities are generally not yet sufficiently numerous to provide reliable results for harmonics higher than the second.

But this local rotation raises as many difficulties as it solves. Our knowledge concerning any condensation of stars in the Sun's neighbourhood is very vague. It rests practically entirely on star-counts which are incapable

* *Harvard Circular*, 350, 1930.

of giving more than the broadest features of actual space distribution. In order to explain the behaviour of the more distant O and B stars and the interstellar calcium observed in their spectra, we are obliged to retain the galactic rotation about a distant centre and then have the task of explaining how the local cluster retains its identity and is not disintegrated comparatively rapidly by the shearing effect of the larger scale motion. If the local cluster is a distinct dynamical unit superposed on a background of stars belonging to the greater galactic system, we might expect to find definite signs of relative motion between cluster and background stars. No such motion has yet been found. The solar motion relative to B stars more than 1000 parsecs away is not very greatly different from the motion relative to the naked-eye stars.

It has to be borne in mind that the observational data suitable for checking these theories of stellar motions are, as yet, not at all extensive. Radial velocities yield us the most reliable information, but even in the northern hemisphere no large group of stars fainter than apparent magnitude 7·5 has yet been observed systematically in this way, and there are still many stars brighter than this which are not yet investigated. To get velocities for ordinary G and K giants at an average distance 1000 parsecs from the Sun, it would be necessary to observe at about the tenth apparent visual magnitude. With so many brighter objects still requiring attention, an extensive observing programme on such stars with present methods and equipment would be entirely uneconomic. In the southern sky, progress is bound to be much slower, although the need for observations there is still more acute. Even after observing all stars down to the tenth magnitude we shall still be completely in the dark as to the radial velocities of " main series " dwarfs more than 100 parsecs from us, and these last, be it remembered, are to the best of our knowledge much more numerous in space than the giants of similar spectral type.

When these observational limitations are considered it becomes clear that, at least with radial velocities, we can at present only test simple theories which must explain the broad, important features of stellar motions, while perhaps saying little or nothing of the more complicated details. In this way the galactic rotation idea, which Lindblad and Oort have developed, has proved very successful, although, as we have seen, the observations of nearer stars do not fit in with it as well as could be desired.

The writer wishes to express his thanks to Dr. J. S. Plaskett for the interest he has taken in this work. It is a pleasure to record the kindness of Dr. C. S. Beals, who helped with some of the observations, particularly in the difficult $5^h - 6^h$ region. A complete account of this work, together with a complete list of the measured velocities, is in press and will appear in due course in the Publications of the Dominion Astrophysical Observatory, Victoria, B.C.

Solar Physics Observatory,
Cambridge :
1931 *December.*

NOTE ON THREE VARIABLE STARS: (i) SV URSÆ MAJORIS, (ii) V URSÆ MINORIS, (iii) WY GEMINORUM.

R. O. Redman, M.A., Ph.D.

(i) *SV Ursæ Majoris* [B.D. $+55^\circ$ 1409, $10^h\ 40^m{\cdot}4$, $+55^\circ\ 34'$ (1900)]

This star is one of those variables whose periods fall between those of the Cepheids on the one hand and the long-period variables on the other. Photometric observations have been summarized by Gerasimovič.* The star may be classed as a semi-regular variable; neither the range of light variation nor the period is constant. Light maxima have photographic magnitudes 10·8 to 10·3 and minima from 11·5 to 10·9. As regards period, Gerasimovič represents the observations by the formulæ

$$\text{Maximum} = \text{J.D. } 2420568 + 78^d{\cdot}0E + 33^d{\cdot}0 \sin (8{\cdot}2E + 140^\circ),$$
$$\text{Minimum} = \text{J.D. } 2420547 + 78^d{\cdot}0E + 33^d{\cdot}0 \sin (8{\cdot}2E + 140^\circ).$$

Prager's 1932 Catalogue of Variable Stars gives the period as 75·3 days.

The proper motion is small.

Radial velocities were determined by the writer during the spring of 1930. The Cassegrain spectrograph of the 72-inch reflector at the Dominion Astrophysical Observatory, Victoria, B.C., was used as described in the previous paper. Owing to the faintness of the star, the slit width was ·003 in. instead of the usual ·002.

The spectrum appears to be of type G, so that the wave-lengths derived for K stars † were used. In accordance with the discussion of systematic errors in the velocities of these K stars,‡ which were obtained with the same instrument and by the same observer, it is recommended that a correction +5 km./sec. be applied to all the values in Table I.

Again, using the K star work, we may expect the probable error of a single observation to be between 3·5 and 4 km./sec. It is possible that the errors were rather greater with SV Ursæ Majoris than with the other stars, since it required exposures (around four hours) three or four times as long as they required. The observed scatter in velocities is thus very little greater than would be expected from a constant velocity star.

The spectrum is similar to that of δ Cephei, and looks as though the star were of great absolute luminosity. The hydrogen lines are variable. $H\beta$ and $H\gamma$ were very weak in the first four plates but strong in the remainder. According to Gerasimovič's formula this diminution of intensity occurred

* *Harvard Bulletin*, **869**, 17, 1929.

† *Publ. D.A.O.*, **4**, 328, 1930.

‡ Preceding paper, or for fuller details, *Publ. D.A.O.*, **6**, No. 5 (in press).

near a light minimum, but at the succeeding minimum the hydrogen lines remained strong. It is likely that the star has the first two or three members of the Balmer series sometimes in emission. *Hα* was not included in these observations.

TABLE I

Radial Velocity Observations of SV Ursæ Majoris

Julian Date	Velocity km./sec.	Quality of Plate	Julian Date	Velocity km./sec.	Quality of Plate
2425986·020	− 96	poor	2426080·785	−108	fair
5989·014	− 95	good	6082·824	− 97	poor
5996·962	− 94	good	6097·757	−102	good
5998·960	− 98	fair	6101·750	−104	good
6019·938	−108	good	6103·811	−108	good
6028·878	−105	fair	6108·774	−100	good
6041·860	−104	good	6110·771	−105	good
6052·833	−113	fair	6112·778	− 95	fair
6063·822	−108	fair			

Mean velocity = −102 ± 1 km./sec.

(ii) *V Ursæ Minoris* [H.D. 119227, $13^h\ 36^m{\cdot}9$, $+74^\circ\ 49'$ (1900)]

This semi-regular variable has a spectrum M4 and a period of about 73 days (Prager's Catalogue gives 67·0). Gerasimovič * represents the photometric observations by the formula :

$$\left.\begin{array}{l}\text{Maximum} = \text{J.D. } 2418978{\cdot}3\\ \text{Minimum} = \text{J.D. } 2418945{\cdot}2\end{array}\right\} + 73^d{\cdot}34E - {\cdot}0055E^2.$$

The light range varies between 0·7 and 0·1. The brightest observed photographic magnitude is 8·8, and the faintest 9·9.

The star was observed at Victoria with the same instrument as used for SV Ursæ Majoris. Two plates were taken as well with a camera of greater focal length giving 49 A/mm. at *Hγ*. The wave-lengths used were Merrill's.† The average exposure was rather less than one hour.

Table II gives the radial velocities. There is no sign of regular variation, and the scatter about the mean value is no greater than would be expected with constant true velocity and a probable error of 3·5 or 4 km./sec. per observation. It is significant that the greatest deviation from the mean velocity is given by the plate of poorest quality. The general quality of the photographs is good. No signs of change of spectrum were noticed. It may be observed that, according to Gerasimovič, the star has periods of pronounced quietude, one of which should have occurred about one period before the first of these observations. This may account for the constancy of the velocities.

* *Harvard Circular*, 388, 1, 1929.
† *Ap. J.*, 58, 221, 1923.

Table II

Radial Velocity Observations of V Ursæ Minoris

Julian Date	Velocity km./sec.	Quality	Julian Date	Velocity km./sec.	Quality
2425997·067	−167	good	2426101·840	−180	good
5997·110	−172	good	6101·892	−162	fair
6020·024	−174	fair	6103·917	−168	fair
6020·076	−170	fair	6108·883	−172	good
6035·030	−180	fair (49 A/mm.)	6122·746	−170	good
6042·010	−179	good (49 A/mm.)	6122·801	−174	good
6063·939	−171	fair	6138·783	−168	good
6064·028	−163	fair	6142·759	−173	good
6075·919	−171	fair	6146·751	−164	good
6085·911	−174	good	6148·802	−173	good
6097·850	−160	poor	6161·767	−168	fair

Mean velocity = −171 ± 1 km./sec.

(iii) *WY Geminorum* [H.D. 42474, $6^h\ 5^m{\cdot}8$, $+23°\ 14'$ (1900)]

This star, according to the Henry Draper Catalogue, is of visual magnitude 7·43 and type K5. Its galactic latitude is $+4°$ and thus it was on the observing list of K stars, with which the previous paper deals. According to Prager's 1932 Catalogue of Variable Stars, it is an irregular variable with light range 7·4 to 7·9. The spectrum has been classified as M3*ep* at Mt. Wilson. One plate was taken at Victoria. It showed an absorption spectrum like that of an F or G star, with titanium oxide bands and numerous emission lines. The Mt. Wilson radial velocity is +18·2 km./sec. The Victoria value (corrected) * at J.D. 2426338·783 was +15 km./sec. With this velocity wave-lengths were determined for the principal emission lines and are given in Table III. λλ 4244, 4277 and 4287 are particularly strong. Three of the six were identified with " forbidden " lines of ionized iron (*Fe* II) found by Merrill † in long-period variables. Later it was discovered that all six have been observed in η Carinæ and at least one other star (H.D. 45677) as well.‡

Considering that the WY Geminorum lines were measured on only one plate of very small dispersion, the agreement with the wave-lengths given by Merrill and Spencer Jones must be accounted satisfactory. According to the identifications given by these two investigators all the lines save λ 4068·8 are due to " forbidden " transitions in singly ionized iron. They suggest that this one exception is a nebular line, but since WY Geminorum is a late type variable with no other signs of nebular emission, so far as is known at present, it seems quite possible that λ 4068·8 is also due to ionized iron.

* See preceding paper.

† *Mt. Wilson Contributions*, **399**, 210, 1930.

‡ Merrill, *Ap. J.*, **67**, 392, 408, 1928; Spencer Jones, *M.N.*, **91**, 794, 1931.

TABLE III

Emission Line Wave-lengths

WY Geminorum	Long-period variables (Merrill)	η Carinæ (Spencer Jones)	
		Observed	Computed
4068·8	...	4068·7	4068·6 (Neb. ?)
4244·5	4244·08	4244·2	4244·0 (*Fe* II)
			4244·8 ,,
4277·4	...	4276·9	4276·9 ,,
4287·6	4287·56	4287·4	4287·4 ,,
4320·0	...	4320·0	4319·6 ,,
4359·2	4359·21	4359·1	4358·4 ,,
			4359·3 ,,

NOTE ON THE EFFECT OF PROPER MOTION ON DOUBLE STAR MEASURES.

Alan Fletcher, M.A.

The scale on which we see any binary system is changing by the fraction $-1{\cdot}02PV \times 10^{-6}$ annually, where P'' is the parallax of the system and V km./sec. is its radial velocity away from the Sun ; hence the small correction Δs, to be applied to the distance s measured at time (in years) t to reduce it to the scale of time t_0, is given by

$$\Delta s/s = +1{\cdot}02PV(t - t_0) \times 10^{-6}. \qquad (1)$$

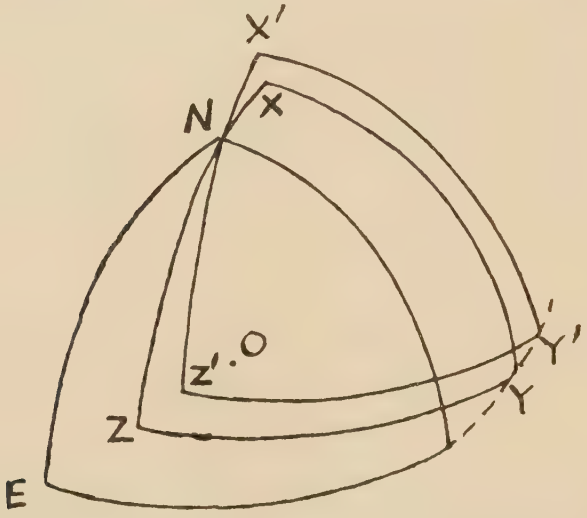
The angle p needs the usual precession correction

$$\Delta p = -0^\circ{\cdot}0056 \sin \alpha \sec \delta (t - t_0). \qquad (2)$$

Let us now consider the effect of proper motion.* For the moment we refer our directions to a fixed equinox, and consider a binary system having proper motion but negligible change of scale. On a celestial sphere with centre O, north pole N and equinox E, let Z represent the position of the system at time t_0 ; find X and Y by travelling 90° from Z in the directions of increasing declination (position angle 0°) and increasing R.A. (position angle 90°) respectively, so that OX, OY and OZ are mutually perpendicular, Y lies on the equator and X, Z on a great circle through N. Let X', Y', Z' be the corresponding points at time t. Then the displacement of the companion relative to the primary is a vector, of which we observe at time t

* Inspired apparently by α Centauri, v. d. Bos has worked out, *via* a transformation of co-ordinates, formulæ convenient for computing the changes to be anticipated in well-determined elliptic elements over periods of time which may be vast (*Union Circular*, No. 68, 1926). The effect of proper motion on the measured co-ordinates during a few centuries is not evident without further work, and for several reasons it is believed that the present simpler investigation, suggested by the considerably different case of 61 Cygni, may also be found useful.

the components ($x' = s \cos p$, $y' = s \sin p$), on a certain scale, in the directions OX', OY' ; but the quantities which satisfy the ordinary dynamical equations are its components (x, y, z), on the same scale, in the fixed directions OX, OY, OZ. Now, if we denote by ($+\Delta\alpha$, $+\Delta\delta$) the small changes of R.A. and dec. which move Z into coincidence with Z', then the small rotations



which move (X, Y, Z) into coincidence with (X', Y', Z') are, with the usual conventions as to sign,

$$-\Delta\alpha \text{ about } N, \qquad +\Delta\delta \text{ about } Y ;$$

i.e.

$$-\Delta\alpha \cos\delta \text{ about } X, \qquad +\Delta\delta \text{ about } Y, \qquad -\Delta\alpha \sin\delta \text{ about } Z.$$

Hence

$$x' = x - y\Delta\alpha \sin\delta - z\Delta\delta,$$

$$y' = y + x\Delta\alpha \sin\delta - z\Delta\alpha \cos\delta ;$$

or, replacing y by y' in the first equation and x by x' in the second,

$$\left.\begin{aligned} x' + y'\Delta\alpha \sin\delta &= x - z\Delta\delta, \\ y' - x'\Delta\alpha \sin\delta &= y - z\Delta\alpha \cos\delta \end{aligned}\right\}. \qquad (3)$$

But $\Delta\alpha \sin\delta = \mu_\alpha \sin\delta(t - t_0)$, where μ_α is the proper motion of the system in R.A., and $\Delta\delta = \mu_x(t - t_0)$, $\Delta\alpha \cos\delta = \mu_y(t - t_0)$, where μ_x, μ_y are the components of its proper motion in the directions of increasing *declination and right ascension respectively*. Also the quantities on the left in (3) equal $s \cos(p + \Delta' p)$, $s \sin(p + \Delta' p)$, where $\Delta' p = -\Delta\alpha \sin\delta$, and may therefore be derived from the observations after applying to the position angles the usual correction *

$$\Delta' p = -\mu_\alpha \sin\delta(t - t_0). \qquad (4)$$

* The corrections Δp and $\Delta' p$ are given by Aitken, *The Binary Stars*, p. 68, 1918, except for a minus sign which needs to be inserted in the second correction.

Denoting these derived quantities by ξ, η respectively, we have from (3)

$$\left.\begin{aligned}\xi &= x - \mu_x(t-t_0)z\\ \eta &= y - \mu_y(t-t_0)z\end{aligned}\right\}. \tag{5}$$

Returning now to the general case, we see that the three corrections (1), (2) and (4) may be applied to the observed (s, p), giving

$$\rho = s + \Delta s, \tag{6}$$

$$\theta = p + \Delta p + \Delta' p. \tag{7}$$

We may then calculate

$$\xi = \rho \cos\theta, \qquad \eta = \rho \sin\theta. \tag{8}$$

Finally, (5) shows that the difference between (ξ, η) and (x, y) depends on z, so that its calculation is mixed up with the determination of the orbit. Several points are to be noticed. First, z is measured, like x and y, in seconds of arc, and is supposed positive when the companion is on the far side of the plane of projection through the primary. Secondly, μ_x and μ_y in (5) are supposed measured in radians. Thirdly, an equation of form (5) applies to every direction in the plane of projection.* Fourthly, the two corrections for proper motion enter through equations (3) on an equal footing, and for stars not too near the pole are in general of the same order of magnitude, so that when the correction which can easily be applied is appreciable, the correction which it is not always easy to apply is likely also to be appreciable.

During a few centuries the effects of light-time, both inside the system and between the system and the earth, are usually negligible; for the general case, see v. d. Bos (*loc. cit.*).

THE BINARY SYSTEM 61 CYGNI.

Alan Fletcher, M.A.

§ 1. The last full discussion of 61 Cygni in its double-star aspect appears to be that of Bergstrand † in 1905; it is by his quadratic formulæ that meridian positions are corrected for orbital motion.‡ These formulæ still represent the observed distances very well, but the observed angles have steadily fallen behind the predicted values ever since 1905, and are now in defect by nearly 1°, equivalent to about 0″·4 of arc. Moreover, Baize § has recently pointed out that the observations of the present century finally dispose of the suggestion that the two components exhibit rectilinear relative

* In the case of 61 Cygni (see p. 123) we shall use only the equations appropriate to the directions of p.a. 117°·60, 207°·60.

† *Nova Acta Regiæ Societatis Scientiarum Upsaliensis*, ser. iv, vol. i, n. 3, 1905, where references to earlier work are given.

‡ *Ast. Papers Amer. Ephemeris*, **10**, pt. 1, p. 147, 1925.

§ *Bull. Soc. Ast. France*, **41**, 20–25, 1927.

motion and are not physically connected. It is now abundantly clear, not only that the system is genuinely binary, but that its period is of the order of a thousand years (probably less), and not of a greater order. Elliptic elements have been found by Baize from a graphical treatment of a considerable fraction of the existing material ; but in the case of such a wide pair it is doubtful how far graphical methods do justice to the unusually high percentage accuracy of the observations, and Baize does not stress the results. Yet a full discussion seems desirable ; for binary systems of such long period form a class of astronomical objects about which observation at the moment gives little information, and it is in those specimens which have very large parallax that we may expect the accuracy of the observations first to outweigh the shortness of the available arc. This arc, even in the case of 61 Cygni, is still too short to permit the evaluation of elliptic elements of real value, particularly for the purpose of deducing the total mass of the system ; on the other hand, if a value of the mass be assumed, the elements then deducible are not very sensitive to changes in the assumed value. Thus a value of the mass based on the approximate mass-luminosity relationship, used, of course, in conjunction with the known parallax of the system, is sufficiently accurate to throw considerable light on the values of the elements, and such a value of the mass has, in fact, been assumed from the beginning. One result of the investigation is, though not very definite, almost independent of the value assumed for the mass ; I think the observational evidence almost alone already tends to suggest that the eccentricity of the orbit is not likely to be very small. But a decidedly better determination by some different method should be possible in twenty or thirty years' time ; whenever a re-determination is made, it is hoped that the labour may be appreciably lightened by the work which has been done in collecting and sifting an almost embarrassing amount of observational material.

§ 2. Several corrections to the measures are necessary (see note, p. 119, in this number). The parallax of 61 Cygni has been assumed to be $0''{\cdot}300$, and its radial velocity -63 km./sec. The zero moment has been taken to be $t_0 = 1880{\cdot}0$ for all purposes. Thus

$$\rho = s + \Delta s, \quad \text{where} \quad \Delta s/s = -0{\cdot}000019(t - 1880). \tag{1}$$

We shall find it convenient to decrease all position angles by $117^\circ{\cdot}60$, the approximate p.a. at 1880·0. We have $\alpha = 315^\circ{\cdot}38$, $\delta = 38^\circ{\cdot}16$, $\mu_\alpha = 0^\circ{\cdot}0015$, $\Delta p = +0^\circ{\cdot}0050(t - 1880)$, $\Delta' p = -0^\circ{\cdot}0009(t - 1880)$, and we shall use

$$\theta = p + \Delta p + \Delta' p - 117^\circ{\cdot}60, \quad \text{where} \quad \Delta p + \Delta' p = +0^\circ{\cdot}0041(t - 1880). \tag{2}$$

Weighted measures of s and p are given, with explanation of their derivation, in Table III at the end of this paper. They are means, in many cases taken over a fairly long period, so that many observers are represented by a single entry and few by more than two. In general, the same *total* weight 12 has been assigned to all apparently good and adequately equipped observers who have made enough measures (say on 15 to 20 nights) to make their error probably largely systematic ; in a few cases slightly greater total weight has been assigned to well-known observers who have measured over a very long

period or with several telescopes, and slightly less total weight to each observer when several have used the same telescope ; less total weight has been assigned to measures made on a small number of nights. Other relevant considerations have been taken into account, and special attention has been paid to the probable accuracy of the distances, on which an orbit deduced from a short arc vitally depends. In this respect the modern photographic measures provide a problem. Omitting a few sets of small weight, their distance residuals from the final ephemeris given in Table II are :—

Observer	Date	Plates	O − C
			″
Wilsing	1891·70	21	−0·070
Greenwich Obs.	1899·80	4	−0·064
Bergstrand	1901·66	53	−0·045
Miss Hopkins I	1904·60	2	−0·013
,, ,, II	1912·90	8	−0·050
Hertzsprung	1916·45	21	−0·049
Przybyllok and Labitzke	1925·95	45	−0·026

Thus the modern photographic distances are rather less than 0″·05 smaller than the ephemeris distances ; or, having regard to the fact that they have been allowed to contribute nearly a quarter of the weight in the modern part of the ephemeris, about 0″·06 smaller than the visual distances. The photographic measures may be more accurate, but they are not on that account the more suitable for determining, in conjunction with the early visual measures, such quantities as the curvature of the apparent relative path of the two components ; consequently the result of a good series of photographic measures has been assigned weight 12 and no more. The difference found above is not due to the comparatively large number of modern visual observations of low weight made and used, for a difference of about the same amount was first suspected from the major series of visual measures alone ; but for this fact the minor series would probably not have been collected at all. The very early photographic measures of 1873·55 by Rutherfurd and Davis stand apart from the modern ones, for they show a residual of +0″·043, but they also show a discrepancy of about 0″·13 between the results of plates taken in positive and negative hour angles.

From Table III were derived 30 weighted normal positions (s, p) ; from these, by (1) and (2), the positions (ρ, θ) given in Table I ; and from these, by

$$\xi = \rho \cos \theta, \qquad \eta = \rho \sin \theta, \tag{3}$$

the positions (ξ, η) also given in Table I. It was found convenient to work with a time-unit of 50 years, and Table I gives the value τ found from $50\tau = t - 1880$. The components of proper motion, μ_x and μ_y, in the directions of p.a. 117°·60 and 207°·60 equal +0·00054 and −0·00114 radians per 50 years respectively, so that (see equations (5), p. 121)

$$\left.\begin{aligned} \xi &= x - 0{\cdot}00054\tau z \\ \eta &= y + 0{\cdot}00114\tau z \end{aligned}\right\}. \tag{4}$$

TABLE I

(Unit for O − C = 0″·001)

t	τ	Wt.	ρ	θ	ξ	O − C	η	O − C
			″	°	″		″	
1834·80	−0·9040	23	15·909	−24·60	14·465	− 10	−6·623	+ 6
38·55	−0·8290	5	16·263	−21·88	15·092	+ 85	−6·061	+37
42·16	−0·7568	12	16·516	−19·81	15·539	+ 33	−5·597	−16
44·93	−0·7014	3	16·706	−17·94	15·894	+ 14	−5·146	+36
49·66	−0·6068	4	17·106	−15·05	16·519	+ 18	−4·442	+53
52·47	−0·5506	5	17·252	−13·70	16·761	− 99	−4·086	0
56·55	−0·4690	15	17·671	−11·26	17·331	− 37	−3·450	+37
59·58	−0·4084	4	17·961	− 9·72	17·703	− 31	−3·032	+ 7
62·03	−0·3594	7	18·157	− 8·43	17·961	− 63	−2·662	+15
65·79	−0·2842	9	18·543	− 6·73	18·415	− 43	−2·173	−54
68·54	−0·2292	2	18·750	− 5·41	18·666	−100	−1·768	−59
71·59	−0·1682	18	19·140	− 3·76	19·099	− 1	−1·255	0
73·86	−0·1228	20	19·422	− 2·77	19·399	+ 56	−0·939	−22
76·90	−0·0620	12	19·695	− 1·48	19·688	+ 28	−0·509	−47
79·64	−0·0072	17	19·996	− 0·15	19·996	+ 57	−0·052	+ 1
82·81	+0·0562	14	20·271	+ 1·17	20·267	+ 14	+0·414	− 7
88·93	+0·1786	17	20·882	+ 3·52	20·843	+ 11	+1·282	−53
91·22	+0·2244	32	21·073	+ 4·61	21·005	− 35	+1·694	+18
94·82	+0·2964	17	21·486	+ 6·02	21·368	+ 11	+2·253	+42
1900·70	+0·4140	26	22·021	+ 8·03	21·805	− 44	+3·076	− 8
03·39	+0·4678	27	22·376	+ 9·02	22·099	+ 35	+3·508	+26
06·25	+0·5250	21	22·605	+ 9·91	22·268	− 17	+3·890	−14
09·33	+0·5866	27	22·928	+10·97	22·509	− 6	+4·363	+ 6
11·72	+0·6344	15	23·208	+11·46	22·745	+ 58	+4·611	−96
13·28	+0·6656	16	23·316	+12·26	22·784	− 13	+4·951	+16
16·03	+0·7206	34	23·577	+13·03	22·970	− 16	+5·316	−19
20·60	+0·8120	13	24·142	+14·34	23·390	+105	+5·979	−21
22·64	+0·8528	25	24·207	+15·19	23·361	− 51	+6·343	+48
26·14	+0·9228	26	24·571	+16·16	23·600	− 22	+6·839	+41
30·37	+1·0074	6	25·066	+16·95	23·977	+116	+7·308	−95

§ 3. To obtain a general idea of the process used in determining the orbit, let us for the moment neglect the small z-terms in (4), and so consider our positions (ξ, η) to be positions (x, y). The variations of x, y, z with the time may be expressed by power series such as

$$x = x_0 + x_0'\tau + \tfrac{1}{2}x_0''\tau^2 + \tfrac{1}{6}x_0'''\tau^3 + \tfrac{1}{24}x_0''''\tau^4 + \cdots \tag{5}$$

where the zero suffix indicates values at time $\tau = 0$, and dashes denote differentiation with respect to τ. The values of x_0'', x_0''', x_0'''' are given by the usual expressions

$$x_0'' = -k^2 x_0 / r_0^3, \tag{6}$$

$$x_0''' = 3k^2 x_0 r_0' / r_0^4 - k^2 x_0' / r_0^3, \tag{7}$$

$$x_0'''' = 6k^2 x_0' r_0' / r_0^4 + \text{terms with factor } x_0, \tag{8}$$

where k^2 is the apparent constant of attraction for the system, and

$$r_0^2 = x_0^2 + y_0^2 + z_0^2. \tag{9}$$

Thus, if we assume a value for k^2, and can find x_0, x_0', x_0'', x_0''' from the x's, we can find r_0 and r_0' successively from (6) and (7). Then, since y_0, y_0' are known fairly accurately, (9) gives z_0, apart from an ambiguity of sign, and z_0z_0' can be found from

$$r_0r_0' = x_0x_0' + y_0y_0' + z_0z_0'. \qquad (10)$$

The y's would provide another determination ; actually it is best to throw practically all the weight of the determination of r_0 and r_0' into one set of equations, by choosing the x-axis as nearly as possible in the direction of the observed position angle at time $\tau = 0$; this makes y_0 nearly zero, and, as mentioned in § 2, has been done. When x_0, y_0, $\pm z_0$, x_0', y_0', z_0z_0' have been found, elliptic elements may be worked out by well-known formulæ.

§ 4. The components of 61 Cygni are of visual magnitudes 5·57, 6·28 apparent ; for parallax 0″·300, $7^{m}\cdot96$, $8^{m}\cdot67$ absolute. Their spectral types are about K7 or 8 and K8. For such stars the mass-luminosity relationship as given by Eddington * suggests masses about 0·62, 0·51, or total mass 1·13⊙. Consequently, all the calculations have been based on an apparent constant of attraction 1·2 seconds of arc^3/years^2 for the system ; this corresponds, for parallax 0″·300, to a total mass 1·126⊙. In our 50-year units the apparent constant of attraction is

$$k^2 = 1\cdot2 \times 50^2 = 3000. \qquad (11)$$

§ 5. A least-squares solution gives

$$\xi = 19''\cdot975 + 5''\cdot005\tau - 1''\cdot168\tau^2 + 0''\cdot029\tau^3, \qquad (12)$$

with probable errors : of the coefficients, 0″·009, 0″·026, 0″·020, 0″·040 ; for unit weight, 0″·120 ; for weight 12, 0″·035. We shall now regard the first three coefficients and the assumed value of k^2 as exact, but shall retain the p.e. of the fourth coefficient ; it is therefore to be understood that the p.e.'s given below in this and the next section require to be slightly increased. It might be thought that our result $+0''\cdot029 \pm 0''\cdot040$ for the fourth coefficient is quite valueless, but this is not the case. Let us for the moment neglect the z-terms of (4), and so identify the above coefficients with x_0, x_0', $\frac{1}{2}x_0''$, $\frac{1}{6}x_0'''$ respectively. Then, if the orbit were circular, we should have, putting $r_0' = 0$ in (7) and then dividing by (6), $x_0''' = x_0'x_0''/x_0$, or, with the above values, $\frac{1}{6}x_0''' = -0''\cdot098$, independently of any assumption about k^2, which the division eliminates. Thus the difference between the value found and the circular value is $+0''\cdot127 \pm 0''\cdot040$, which suggests † that the stars are less

* *The Internal Constitution of the Stars*, chaps. vi and vii, 1926. The effective temperatures have been assumed to be 3450° and 3400°, in line with the values used on p. 154, and the correction suggested in the footnote on p. 159 has been made.

† A least-squares solution has been made because it is a mechanical and unbiased process ; unfortunately, the probable errors which emerge are not necessarily entitled to be regarded as accurate indices to the corresponding probabilities. In view of this fact, it may be as well to say that a small positive coefficient of τ^3 was expected before the solution was made, and I am not sure that there is any reasonable method of analysing my material finely which would include the circular value among those most highly probable. Nevertheless, the orbit may actually be nearly circular, like the one that Baize found graphically.

likely to be describing a nearly circular orbit than to have been moving in 1880 from periastron to apastron in a decidedly elliptic one ; for we shall see that if the value $+0''\cdot127$ were actually correct, then the eccentricity of the orbit (though this does depend a little on k^2) would be about 0·4.

§ 6. Let us now, still identifying ξ with x, work out approximate values of z_0 and z_0'. (6) gives $r_0 = 29''\cdot49$, then (9), with $y_0 = 0$, gives $z_0 = 21''\cdot69\epsilon$, where $\epsilon = \pm 1$. Also $\frac{1}{2}z_0'' = (z_0/x_0) \times \frac{1}{2}x_0'' = -1''\cdot27\epsilon$. Dividing (7) by (6) we find

$$\frac{r_0'}{r_0} = \tfrac{1}{3}\left(\frac{x_0'}{x_0} - \frac{x_0'''}{x_0''}\right), \tag{13}$$

or, with the above values, $r_0'/r_0 = 0\cdot107 \pm 0\cdot034$. Then (10), with $y_0y_0' = 0$, gives $z_0' = -0''\cdot33\epsilon \pm 1''\cdot37$. Thus

$$z = \epsilon(21''\cdot69 - 0''\cdot33\tau - 1''\cdot27\tau^2 \pm 1''\cdot37\tau),$$
$$z' = \epsilon(-0''\cdot33 - 2''\cdot54\tau \pm 1''\cdot37),$$

or, in terms of radial velocity, ($1''$/50 yr. = 0·316 km./sec.),

$$z' = \epsilon(-0\cdot10 - 0\cdot80\tau \pm 0\cdot43) \text{ km./sec.}$$

As might be expected from the smallness of the quantities involved, the relative radial velocity evidence * is not too certain, but we are compelled to make an inference from it. It seems that around 1920 ($\tau = 0\cdot8$) the relative radial velocity was perhaps something like +2 km./sec. (companion receding relative to primary). This points to the value $\epsilon = -1$, which is probably correct and has been used ; if ϵ actually $= +1$, then the small z-corrections will have been applied below with the wrong sign, though only to x_0', y_0' will perceptible injury have been done. Using, then, $z = -21''\cdot69 + 0''\cdot33\tau + 1''\cdot27\tau^2$, we have $0\cdot00054\tau z = -0''\cdot012\tau + 0''\cdot001\tau^3$; applying this to the ξ of (12), we have from (4),

$$x = 19''\cdot975 + 4''\cdot993\tau - 1''\cdot168\tau^2 + 0''\cdot030\tau^3.$$

This value is final ; for on repeating the above calculation, taking $\epsilon = -1$, we find that $z = -21''\cdot69 + 0''\cdot23\tau + 1''\cdot27\tau^2$, and $0\cdot00054\tau z$ requires no change. It will be noticed that the slight changes in the values of x_0' and $\frac{1}{6}x_0'''$ scarcely affect the argument of § 5.

§ 7. Let us now consider the η's and y's. By a suitable choice of axes we have taken y_0 to be so small that we may equate it to zero in (6), (7) and (8). Then (5) gives

$$y = y_0 + y_0'\tau - (\tfrac{1}{6}k^2y_0'/r_0^3)\tau^3 + (\tfrac{1}{4}k^2y_0'r_0'/r_0^4)\tau^4.$$

Assuming the values of k^2/r_0^3 and k^2r_0'/r_0^4 indicated by § 6, and the sufficiently accurate preliminary value $7''\cdot5$ for y_0', we find

$$y = y_0 + y_0'\tau - 0''\cdot146\tau^3 + 0''\cdot024\tau^4.$$

Also

$$0\cdot00114\tau z = -0''\cdot025\tau + 0''\cdot001\tau^3.$$

* Adams and Joy, *Ap. J.*, **57**, 172, 1923 ; Harper, *Pub. D.A.O.*, **2**, 200, 1924. See also *Lick Obs. Bull.*, 214, 1912. The value +2 km./sec. corresponds to a nearly circular orbit, but this indication is of comparatively low weight.

Hence, by (4),

$$\eta = y_0 + (y_0' - 0''\cdot025)\tau - 0''\cdot145\tau^3 + 0''\cdot024\tau^4.$$

A least-squares solution to express the quantity $\eta + 0''\cdot145\tau^3 - 0''\cdot024\tau^4$ as a linear function of τ indicates $y_0 = +0''\cdot001 \pm 0''\cdot005$, $y_0' = +7''\cdot495 \pm 0''\cdot009$; p.e.'s: for unit weight, $0''\cdot098$; for weight 12, $0''\cdot028$. It will be noticed that the η's, depending chiefly on the position angles, are markedly more accurate than the ξ's, depending chiefly on the distances. The slightly peculiar distribution of the early residuals (mainly of low weight) in both ξ and η is largely fortuitous; the early distance measures are very scattered, and any attempt to follow the run of the residuals would result in satisfying the early normal places far too well.

§ 8. If we assume that $k^2 = 3000$,

$$\begin{aligned} x_0 &= +19''\cdot975, & x_0' &= +4''\cdot993, \\ y_0 &= +\ 0''\cdot001, & y_0' &= +7''\cdot495, \\ z_0 &= -21''\cdot69, & z_0' &= +0''\cdot23, \end{aligned}$$

we can evaluate a set of elliptic elements. They must not be used to deduce the mass of the system, for a value of the mass has been assumed. The values obtained for individual elements cannot be stressed, but the set as a whole contains only one degree of really rough approximation.

It is clear from § 6 that the difficulty about the sign of the inclination is more than usually annoying; there is little doubt that the stars are now approaching a node, and a certain knowledge of the *sign* of the relative radial velocity is very desirable.

The results of the computation are :—

$a = 24''\cdot525$	Ω, referred to p.a. $117^\circ\cdot60$, $= 55^\circ\cdot223$.
$e = 0\cdot40369$	Motion direct.
$P = 696\cdot63$ years	$M = 0^\circ\cdot51677(t - 1686\cdot37)$
$T = 1686\cdot37$	$= E - 23^\circ\cdot130 \sin E$.
$i = +52^\circ\cdot896$	$x = -17''\cdot907 \cos E + 4''\cdot359 \sin E + 7''\cdot229$.
$\omega = 153^\circ\cdot917$	$y = -14''\cdot383 \cos E - 15''\cdot038 \sin E + 5''\cdot806$.
$\Omega = 172^\circ\cdot823$	$z = -1\cdot086x + 0\cdot754y$.

These elements are referred to the scale, equinox and view-point of 1880. The equations for deducing apparent distance s and position angle p are $\tau = \frac{1}{50}(t - 1880)$, (4), (3), (1) and (2). Values may, however, be found by second-difference interpolation throughout the following ephemeris.

TABLE II

t	s	p	t	s	p
	″	°		″	°
1820	14·979	82·62	1900	22·004	125·31
1830	15·576	89·91	1910	23·013	128·65
1840	16·300	96·60	1920	24·005	131·70
1850	17·123	102·68	1930	24·972	134·52
1860	18·025	108·18	1940	25·908	137·13
1870	18·983	113·13	1950	26·805	139·56
1880	19·975	117·60	1960	27·659	141·83
1890	20·987	121·64			

The above positions are computed from the elliptic elements. From 1840 to 1920 inclusive they are identical with positions computed from the power series. For 1830 and 1930 the differences in ξ and η do not exceed 0″·006. The full form of (8) suggests for the coefficient of τ^4, omitted in ξ, the negligible value of a few thousandths of a second.

§ 9. Some account of the material used has already been given in § 2. Table III makes no pretence to be exhaustive, but it is hoped that little material of value has been omitted. In particular, all observations mentioned by C. F. W. Peters * or Bergstrand (*loc. cit.*) or in Burnham's *General Catalogue*, or traceable from the appendix to Innes's *Reference Catalogue* or the Council Reports of the R.A.S., have been considered for inclusion. Very early observations have not been used. Most of the entries are simple means from a number of visual measures (v.) or photographic plates (p.) which is indicated, along with the approximate aperture of the telescope used, in the final column. It is the number of distance measures which is given ; isolated angle measures have not been used, except in a few cases in which the total material is scanty. When the entry has been derived from published means, the accuracy of these means has been checked.

Some of the means were taken over a considerable number of years, and to these corrections never exceeding 0″·006 and 0°·04 have been applied. It is obvious that if an observed quantity is not varying linearly with the time t, but contains a term ct^2, then the ordinary weighted mean, to be valid for the weighted mean time $\bar{t}$, requires a correction $-kc$, where k is the weighted mean of $(t-\bar{t})^2$. The value of k was computed in all cases in which measures over more than two or three years were combined. Some preliminary work suggested that it would be sufficient in the case of the distances to make $c=\frac{1}{2}\ddot{s}$ run linearly from $+0''\cdot00055$ (years^{-2}) in 1830 to $-0''\cdot00025$ in 1930, and this was done ; actually this representation turns out to be, though so much was not demanded of it, not far from the best possible linear one, judging by the second differences derivable from Table II. In dealing with the angles we are on safe ground, for $\ddot{\theta}=-2\dot{\rho}\dot{\theta}/\rho$, and $c'=\frac{1}{2}\ddot{p}$ can be computed with sufficient accuracy without any knowledge of the orbit. It was considered sufficient to make c' run linearly from $-0°\cdot0033$ in 1830 to $-0°\cdot0010$ in 1930. The corrections are mentioned here, not so much because they are important in themselves, as in order to make it clear that due caution has been exercised in taking means of long series of observations.

I wish to acknowledge my indebtedness to Dr. W. M. Smart for valuable encouragement and advice.

TABLE III

Observer	t	s	p	Wt.	Remarks
		″	°		
Dawes I	1833·69	15·950	92·54	1	4 in., 6 v.
Bessel	34·80	15·940	92·97	8	6 in., 22 v.
W. Struve	34·82	15·865	93·34	12	9 in., 19 v.
W. H. Smyth	35·19	15·863	93·53	2	6 in., 6 v.

* *A.N.*, **113**, 2708–9, 1886.

Observer	t	s	p	Wt.	Remarks
		″	°		
Encke	1838·55	16·250	95·89	5	9 in., 6 v.
Dawes II	41·22	16·485	97·35	4	7 in., 6 v.
Challis and Glaisher	41·99	16·536	97·50	2	12 in., 9 v.
Schlüter	42·84	16·506	98·50	6	6 in., 16 v.
Mädler I	43·79	16·481	99·49	1	9 in., 9 v.+
O. Struve I	45·50	16·802	99·95	2	15 in., 6 v.
Dawes III	49·66	17·096	102·67	4	6 in., 7 v.
O. Struve II	52·21	17·335	103·87	3	15 in., 9 v.
Mädler II	52·85	17·105	104·21	2	9 in., 39 v.
Morton	55·69	17·521	105·89	7	8 in., 7 v.
Dembowski I	56·75	17·566	106·72	3	5 in., 19 v.
Jacob	57·63	17·919	107·04	5	6 in., 9 v.
O. Struve III	59·36	18·087	107·81	2	15 in., 5 v.
Mädler III	59·80	17·820	108·12	2	9 in., 34 v.
Auwers	61·36	18·083	109·08	4	6 in., 8 v.
Romberg	62·93	18·241	109·46	3	10 in., 3 v.
Main I	65·10	18·359	110·36	3	7½ in., 11 v.
Dembowski II	66·14	18·628	111·22	6	7 in., 58 v.
O. Struve IV	68·54	18·746	112·24	2	15 in., 4 v.
Knott	70·61	19·175	113·25	3	7 in., 7 v.
Dunér	71·22	18·974	113·64	8	10 in., 25 v.
Gledhill I	72·44	19·306	114·40	7	9⅓ in., 9 v.
Main II	73·54	19·381	114·16	4	7½ in., 23 v.
Rutherfurd and Davis	73·55	19·375	114·82	10	13 in., 19 p.
Dembowski III	74·59	19·521	115·38	6	7 in., 60 v.
Temple Obs. I	75·80	19·594	115·95	3	8 in., 12 v.
O. Struve V	76·26	19·642	116·03	3	15 in., 8 v.
Plummer	77·77	19·774	116·31	4	12 in., 7 v.
Flammarion	77·79	19·76	116·20	2	15 in., 3? v.
Goldney	78·73	20·022	116·70	2	6 in., 4 v.
Jedrzejewicz	79·45	20·010	117·33	9	6½ in., 22 v.
O. Struve VI	80·07	19·981	118·00	3	15 in., 7 v.
Temple Obs. II	80·40	19·952	117·75	3	8 in., 12 v.
Schiaparelli I	82·43	20·174	118·61	5	8½ in., 11 v.
Hall I	82·71	20·297	118·77	6	26 in., 36 v.
Perrotin	83·64	20·387	118·97	3	15 in., 4 v.
D'Engelhardt	86·92	20·74	120·1	2	12 in., 2 v.
King	89·09	20·795	120·90	3	13 in., 2 p.
Hall II	89·18	20·944	121·21	6	26 in., 37 v.
H. Struve I	89·28	20·922	121·37	6	30 in., 6 v.
Schiaparelli II	90·45	20·930	122·01	10	19 in., 32 v.
Burnham I	90·87	21·145	121·88	3	12 in., 4 v.
Toulouse Obs.	90·89	21·180	122·18	3	9+in., 4 v.
Wilsing	91·70	21·090	(122·31)	12	13 in., 21 p.
Bigourdan	92·18	21·274	122·29	4	12 in., 5 v.
Cohn I	93·23	21·217	122·77	3	6 in., 6 v.
Chofardet and Petit	94·58	21·511	123·60	6	Notes.
Seraphimow	95·52	21·505	123·44	2	15 in., 2 v.
Comstock	95·55	21·580	124·12	4	10 in., 5 v.
Collins	95·77	21·66	123·6	2	10 in., 2 v.
Cohn II	99·54	21·971	125·34	4	13 in., 7 v.
Greenwich Obs.	1899·80	21·92	125·36	8	26 in., 4 p.
Gledhill II	1900·86	22·0	124·9	2	9 in., 7 v.
Bergstrand	01·66	22·127	125·85	12	13 in., 53 p.

Observer	t	s	p	Wt.	Remarks
		"	°		
Barnard I	1902·67	22·361	126·46	8	40 in., 142 v.
H. Struve II	03·40	22·351	126·51	9	13 in., 34 v.
Postelmann	03·54	22·423	126·28	6	13 in., 11 v.
Miss Hopkins I	04·60	22·457	127·04	4	40 in., 2 p.
Burnham II	05·41	22·50	127·2	7	40 in., 8 v.
Morgan	05·84	22·56	127·5	2	12 in., 3 v.
Cambridge Obs. I	05·87	22·58	126·7	2	12½ in., 1 p.
Lau	06·92	22·597	127·50	4	10 in., 2 p.
v. Biesbroeck I	07·06	22·794	127·76	6	15 in., 9 v.
Doberck I	07·98	22·726	128·04	4	Notes.
Barnard II	09·05	22·969	128·47	6	40 in., 16 v.
Hassenstein	09·72	22·852	128·58	3	13 in., 4 v.
Urania Obs.	09·75	23·009	128·53	14	10 in., 26 v.
Fox	10·41	23·225	127·82	3	18½ in., 4 v.
Voûte	11·93	23·202	129·20	10	10½ in., 21 v.
Neujmin	12·61	23·315	129·28	2	15 in., 2 v.
Miss Hopkins II	12·90	23·253	129·66	8	40 in., 8 p.
v. Biesbroeck II	13·63	23·438	129·76	6	Notes.
R. Andersen	13·73	23·325	129·80	2	14 in., 2 v.
Leavenworth I	14·96	23·57	129·5	1	10½ in., 2 v.
Doberck II	15·42	23·529	130·32	4	7 in., 14 v.
Guillaume	15·52	23·62	130·2	2	13 in., =2 v.
Rabe I	15·74	23·512	130·41	8	8 in., 72 v.
Beal I	15·79	23·76	130·2	2	10½ in., 6 v.
Hertzsprung	16·45	23·606	130·60	12	20 in., 21 p.
Pettit	16·46	23·557	131·00	3	11½ in., 3 v.
Brown	16·59	23·83	130·6	2	18½ in., 3 v.
v. Steenwijk	18·69	23·95	131·75	2	10½ in., 3 v.
Phillips	20·71	24·11	132·2	3	8 in., 5 v.
Leavenworth II	20·87	24·19	131·2	2	10½ in., 3 v.
Mme. Chandon	21·09	24·248	131·75	6	12 in., 9 v.
Labitzke	22·09	24·316	132·34	5	13 in., 8 v.
Przybyllok	22·42	24·216	132·69	6	13 in., 11 v.
v. d. Bos	22·62	24·193	132·48	10	10½ in., 24 v.
Kuiper	23·59	24·32	132·84	2	13 in., 1 p.
Shajn	23·78	24·12	133·6	2	13 in., 1 p.
Doberck III	25·22	24·461	133·30	4	7 in., 12 v.
Przybyllok and Labitzke	25·95	24·558	133·33	12	13 in., 45 p.
Rabe II	26·30	24·457	134·04	4	8 in., 11 v.
Commendantoff	26·66	24·66	133·7	2	15 in., 3 v.
Manson	26·83	24·91	134·1	2	12½ in., 3 v.
Buchar	27·64	24·95	133·9	2	8½ in., 5 v.
Beal II	28·64	25·08	133·9	2	10½ in., 4 v.
Cambridge Obs. II	30·63	25·19	134·4	2	12½ in., 1 p.
Fletcher	31·85	25·00	134·73	2	8 in., 3 v.

Notes

(For further references, see Burnham)

Dawes : simple means ; II, 1839·759 ½-wt. Smyth : *Bed. Cat.*, 494 (1844). Encke : *Berlin*, **1**, 142 *ff.*; **3**, 234. Challis and Glaisher : *Cambridge Obs.*, **24**, pt. 1. Schlüter : *Königsberg*, **24**, 69 ; **35**, 254. Mädler : means, various weights, of annual means. O. Struve : " uncorrected," except distances from *Obsns. Poulk.*, **9**,

diminished by 1 in 427; see **9**, (130). MORTON: *Mem. R.A.S.*, **29**. DEMBOWSKI: I, mean 2 : 1 : 2 : 2 : 2 of D.'s 5 means in *S.D.M.*, **1**; II, mean of first four means in *S.D.M.*, **2**; III, mean 2 : 2 : 2 : 2 : 1 of last five means in *S.D.M.*, **2**. JACOB: *Mem. R.A.S.*, **28**. AUWERS: *Königsberg*, **35**, 247. ROMBERG: *Leyton*, **1**; mean 2 : 1 : 2. MAIN: *Radcliffe*, **21** *ff.*; I, simple mean, 11 nights, 1861–8; II, mean 2 : 3 of 2 means for 69–72 and 75. KNOTT: *Mem. R.A.S.*, **43**; mean 2 : 3 : 2 of K.'s 3 means. DUNÉR: mean of D.'s 6 means, except distance 1867·88 rejected, altering 1867·89 mean to 18″·677. GLEDHILL: I, *Mem. R.A.S.*, **42**; II, *M.N.*, **61**, 556. RUTHERFURD and DAVIS: *Columbia Univ. Obs. Contns.*, No. 12; 13 plates in pos. hour angle, 5 in neg., discrepancy o″·13 and o°·35; correction −o″·009, +o°·13 (Bergstrand) applied to mean. TEMPLE OBS.: Wilson, Seabroke and Smith. Measures after 1883 not used (large distance errors, as noticed by Bergstrand). To 1883, *Mem. R.A.S.*, **42, 43, 46, 48**; 24 complete measures after rejecting 81·73, 81·74. PLUMMER: *Oxford Obsns.*, No. 1; 2 measures in 1876 not used. FLAMMARION: *Catalogue*. GOLDNEY: *A.N.*, 2257. JEDRZEJEWICZ: mean of 5 means for 1876, 77, 79, 80, 81. SCHIAPARELLI: I, mean of 3 means in *M.S.D.*, **1**. HALL: amended values, *A.J.*, 258; I, mean of annual (½-wt.) and first three biennial means; II, mean of last three biennial means. KING: *H.C.O. Circ.*, 332. TOULOUSE: mean 2 : 1, Cosserat and Saint-Blancat; *Toulouse Obsns.*, **3**, 124. WILSING: *Potsdam*, **11**, pt. 3; 116 plates, hour angle error. I have used mean of 21 p. taken in 1891 between h.a. −o^h·9 and +o^h·9 inc., and have supplied a pos. angle from Bergstrand's formulæ. BIGOURDAN: *Bull. Ast.*, **12**; mean 3 : 2 of means 1890 and 93–4. CHOFARDET and PETIT: *A.N.*, 3450; 13 in., 5 v. +8 in., 4 v. SERAPHIMOW: *Pulk. Mitt.*, No. 76. COHN II, H. STRUVE II, POSTELMANN, HASSENSTEIN: *Königsberg*, **43**, pt. 2. GREENWICH OBS.: *M.N.*, **60**, 519. BERGSTRAND: means from Table 6, plus $\delta\beta$-corrections: +o″·004 in ξ, −o″·029 in η. BARNARD: I, *A.N.*, 4128; B.'s mean for 1901·0 reduced to mean date 1902·67 by his assumed annual changes. II, *A.N.*, 4128, p. 384; *Allegheny*, **2**, 14. MISS HOPKINS: *Columbia Univ. Obs. Contns.*, No. 29. Mean measured co-ordinates (½-wt. to 2 "fair" plates) were derived from Table II, and the following constants for reduction to equinox 1912·8 were assumed: early plates, $a = +·0019$, $b = +·0170$, $d = -·0163$, $e = +·0017$; late plates, $a = +·0018$, $b = +·0167$, $d = -·0160$, $e = +·0016$. MORGAN: *A.J.*, 584–5. LAU: *A.N.*, 4232; mean 2 : 1 of 2 plates with 10-in. v. BIESBROECK: *Ann. Belg.*, **9, 13, 14**; *Yerkes*, **5**; I, mean of means for 1903, 7, 9; II, mean of means for 1911, 13, 15; 15 in., 7 v. +12 in., 3 v. DOBERCK: *Ast. Ab.*, **7**; *A.N.*, 5716; I, mean of means for 1902, 8, 9, 10; 15 in., 2 v. +7 in., 10 v. II, mean of means for 1911, 14, 16, 18; $s(1914) = 23″·41$, as in *A.N.*, 5058. III, mean of means for 1920, 4, 5, 9. URANIA OBS., Copenhagen: Lau 13 v., Luplau-Janssen 9 v., Buch-Andersen 4 v.; *A.N.*, 4078, 4221, 4279, 4315, 4488, 4735. FOX: *Dearborn*, **1**. VOÛTE: *Leiden*, **10**, pt. 2; mean 1 : 2 : 2 of V.'s means for 1910, 11, 12. NEUJMIN: *Pulk. Mitt.*, No. 88. R. ANDERSEN: *A.N.*, 4747. LEAVENWORTH, BEAL: *Minnesota*, **1**. GUILLAUME: *Lyon Bull.*, **13**, 151. RABE: I, mean 1 : 4 : 4 : 4 : 4 : 2 : 4 : 4 : 2 of R.'s 9 means in *A.N.*, 5205–6; II, *A.N.*, 5738–9. HERTZSPRUNG: *Potsdam*, **24**, pt. 2; mean of H.'s 2 means. PETTIT: *A.J.*, 715. BROWN: *A.J.*, 878. v. STEENWIJK: *M.N.*, **80**, 223. PHILLIPS: *M.N.*, **81**, 566. MME. CHANDON: *Journ. des Obs.*, **9**, 47; ½-wt. 1st and 6th. LABITZKE: *A.N.*, 5364; 1921·693 rejected, perhaps inadvisably. PRZYBYLLOK: *Königsberg*, **44**; mean 5 : 3 of P.'s 2 means; +o″·005 to dist. for scale. v. D. BOS: *Leiden*, **14**, pt. 3; mean 1 : 2 : 2 of B.'s 3 means. KUIPER: *B.A.N.*, 168. SHAJN: *A.N.*, 5516. PRZYBYLLOK and LABITZKE: *Königsberg*, **45**, pt. 4. COMMENDANTOFF: *Pulk. Bull.*, No. 105. MANSON: *A.J.*, 910. BUCHAR: *Mém. Soc. Roy. Sci. Bohême*, 1929, *Sciences*, XVI; mean p.a. 133°·83, B.'s 4°·00; 3°·9 a compromise.

RU AQUARII.

P. M. Ryves.

1. The variability of this star, B.D. – 18° 6299, was suspected by Mrs. Fleming in 1901 on account of its spectrum, M*c* with emission lines, and was confirmed by an examination of numerous photographs. The second catalogue of Harvard College Observatory (1907) gives the range as 8·3 – < 9·4 photographic, no period being indicated.

Observations made at Zaragoza in 1910 revealed four maxima occurring at intervals of approximately 66 days. The first is indicated indefinitely by an observation on June 14 (J.D. 2418837) showing the star brighter than the ninth magnitude, whereas 16 days later it was estimated as just below the 10th mag. Although the brightness was probably overrated on the occasion of the first observation, when the variable was low in the morning twilight, there can be little doubt that a maximum took place about that date or a few days earlier. The three following maxima were adequately observed and well defined. After a seasonal gap of 148 days, during which two maxima may have taken place, another well-defined maximum was observed about 1911 July 16, in good agreement as to phase with those of the previous year. After this the observations become scanty and it is difficult to make concrete deductions from them. Another maximum evidently occurred about September 30 (1911), the interval between this and the previous one being a little longer than the others. This maximum appears to have been flat and ill defined and the curve subsequently flattens out, no further maximum being recognisable up to the end of the season. The range had evidently become smaller, but the observations are so few that nothing can be deduced with certainty.

The Julian days corresponding to the above-mentioned maxima are as follows : 2418893, 2418965, 2419025, 2419235, (2419312).

A period of $69^{d}\cdot5$ was derived from the above maxima ; the minima give 68·0 ; the mean may be taken as $68^{d}\cdot7$. The extreme range or amplitude, excluding one observation, is $1^{m}\cdot3$ (9·0 – 10·3), but after the first minimum it is never greater than $0^{m}\cdot9$; and this is probably the limit within any one cycle.

Hayes had previously suspected a period of about 75^{d} and Hartwig found a period of $64^{d}\cdot6$, but these announcements had not been seen by the writer, so the above conclusion was arrived at independently and in a general way confirms Hartwig's result.

Taking the period as $68^{d}\cdot7$ and computing backwards and forwards I find a reasonably good agreement with the dates of a series of photographically determined maxima of RU Aquarii published in *Harvard Bulletin*, No. 848, to which reference will be made later. The elements adopted,

$$\text{Max.} = \text{J.D. } 2415601 + 68\cdot7E,$$

appear to satisfy existing observations over a considerable number of years (1901–1920), but not during the whole period under review. Some visual observations made in 1922 and following years show a smaller range of variation and many apparent irregularities. Certain points taken as maxima are not in phase with the maxima of previous years. There is some indication of a discontinuity or substitution having taken place about 1920, maxima and minima having possibly changed places ; but in view of the small amplitude of the variation and the lack of adequate observational material this point cannot be decided with certainty. As an explanation of these phenomena it may be suggested that RU Aquarii probably goes through *disturbed periods*, during which the regular periodic variation is in abeyance or is masked by the irregular disturbances, to emerge after a time unchanged, except perhaps for a discontinuity.

The red colour of the variable is a hindrance to observation, and the difficulty is increased by an unfavourable position in the sky as seen from this station. With a range of less than one magnitude the task of disentangling the true periodicity or periodicities, the irregularities, errors of observation and seasonal effects (selective absorption) is by no means an easy one.

2. B. P. Gerasimovič, in *Harvard Bulletin*, No. 848 (1927), has published a study of this star based on an examination of 430 Harvard photographs covering the period 1901–1923. He arrives at the conclusion that RU Aquarii has " a surprisingly peculiar light-curve," and adds, " it is very difficult to find anything similar among the large variety of long-period variables that have been investigated." Briefly, he finds that the light-curve is unsymmetrical, the minima being alternately long and short. The maxima occur in pairs, 65 days apart, whilst 105 days elapse between the second maximum of one pair and the first of the following pair. The maxima are termed " principal " and " secondary," but are both of the same height, and the minima, though of very unequal length, are of the same depth. The curve as a whole is variable as regards total range and median brightness. The maximum range is $0^{m}\cdot7$ and the minimum $0^{m}\cdot3$, both photographic, whilst the median brightness oscillates between $10^{m}\cdot4$ and $10^{m}\cdot7$ *Pg*. The following elements are given, I indicating a " principal " maximum, or first of a pair, and II a " secondary " or second of a pair :—

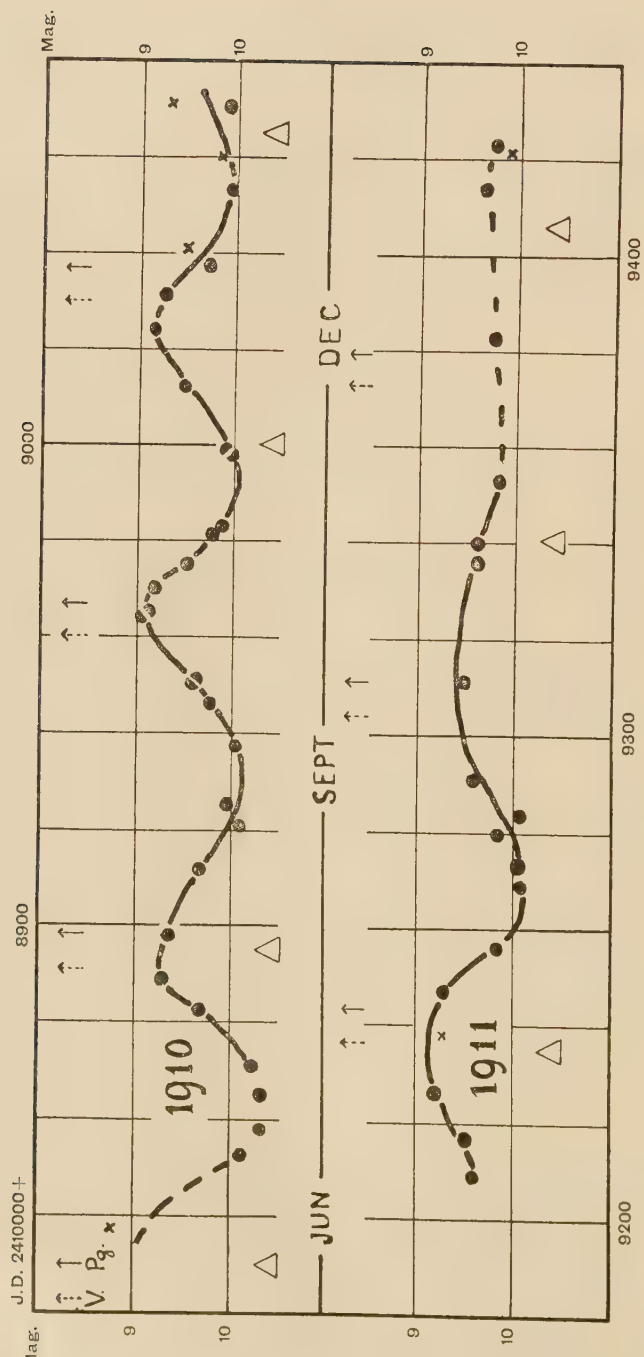
$$\text{Max.} = \left.\begin{matrix}\text{J.D. } 2417808 \text{ I} \\ \text{J.D. } 2417873 \text{ II}\end{matrix}\right\} + 170^{d}\cdot3E \ . \ . \ . \ (1901–1912).$$

After 1912 there are large discrepancies between the dates of observed maxima and those indicated by the above formula. An adjustment is therefore made, it being assumed that a displacement of phase amounting to 54 days has taken place. The new elements are :—

$$\text{Max.} = \left.\begin{matrix}\text{J.D. } 2421160 \text{ (I)} \\ \text{J.D. } 2421225 \text{ (II)}\end{matrix}\right\} + 170^{d}\cdot3E \ . \ . \ . \ (1913–1923).$$

In this way a better agreement is effected, but residuals are still large.

3. The characteristics embodied in the above formulæ indicate a type of



Light-curve of RU Aquarii, 1910–11.

The crosses represent inferior observations. Arrows show dates of maxima according to the elements given in this paper, the visual maxima being assumed to take place 7 days before the photographic. Triangles mark the dates of maxima according to the elements published in Harvard Bulletin, *No. 848, 1927.*

variation quite without precedent, as far as the writer is aware, and would be of very great interest if substantiated. It is therefore unfortunate that the author does not give fuller details as to the observational evidence upon which his conclusions are based, the more so since an examination of the only data published suggests that the foundation is a slender one. Mr. Gerasimovič gives a table of photographically determined maxima, derived from Harvard plates, but nothing to indicate the amplitude, weight or material upon which each was based and no observations or evidence as to the behaviour of the star at other times.

It is obvious from the details given in the first section of this paper, and from the accompanying light-curve, that the observations made at Zaragoza in 1910–11 do not support the supposition of an unsymmetrical light-curve (minima alternately long and short) or a period of 170 days; on the contrary they show definitely that during the season in question three, and almost certainly four, maxima took place at intervals differing little from 66 days. It may be contended that at other times the variation may be of the type suggested, but it is curious that in the list of 26 maxima contained in Mr. Gerasimovič's paper there is no example of a long minimum (105^d) being actually observed. There is one case of what should have been a long minimum, that is a "principal" maximum following a "secondary," but the interval observed (from J.D. 2417043 to . . . 7122) was only 79 days, which is 26 days shorter than that required by the elements, and only 10 days longer than the uniform period found by the present writer.

Again, referring to the table of observed maxima, we see that during the first 8 seasons (1901–1908) two maxima were observed in each year out of the four or five which should occur annually. In the remaining 15 years not more than one maximum was observed in any one year, and there are intervals of 16 and 17 cycles (of period 89 days) without any maximum being observed. Altogether there are, during this time, determinations of the dates of 10 maxima out of the 64 or 80 indicated respectively by the formulæ given in *Harvard Bulletin*, No. 848, and in the present paper. It is obvious that a variety of different formulæ might be made to fit into such scattered data within the rather wide O – C discrepancies shown, especially when an unsymmetrical light-curve is assumed and a displacement of phase about the middle of the period is allowed.

Two mean light-curves accompany Mr. Gerasimovič's paper, each of which shows the type of variation described above—a "secondary" maximum situated unsymmetrically between two "principals"—whilst they differ in range and median brightness. As mean curves arrived at through the assembly of scattered observations in accordance with the adopted elements, their reality is conditional on the correctness of the said assumed elements. Since one complete cycle appears never to have been observed and the intervals without observations are long, they do not carry conviction.

Referring to the second of these curves, which corresponds to the years 1918–1923, we see that out of the 25 or 31 maxima which, according to the respective elements, must have taken place during that lapse of time, only four were observed, three "principal" and one "secondary". It appears,

therefore, that the part of the " mean " curve corresponding to the latter phase rests upon the evidence of a single cycle. The observed date of this maximum is in good agreement with the calculated, but the other three leave residuals of 15 days and more. Two of these are positive, and their mean value $17^{d}\cdot5$ represents a point on the curve actually nearer minimum than maximum. It may be added that the total range of this mean curve is $0^{m}\cdot15$.

4. As stated previously, the uniform period of $68^{d}\cdot7$ derived from the Zaragoza observations (1910–1911) was found to agree reasonably well with the dates of maxima photographically determined at Harvard. In the accompanying table the first column gives number of cycle, the second column the dates of maxima photographically determined at Harvard, the third the O – C residuals resulting from the elements given by Mr. Gerasimovič in *Harvard Bulletin*, No. 848, the line of dots showing where a change is made in the elements, and the last column similar residuals resulting from the adoption of a uniform period of $68^{d}\cdot7$, initial epoch J.D. 2415601. The mean deviations for the last two columns are $9^{d}\cdot4$ and $9^{d}\cdot0$ respectively.

Table I

Cycle	J.D.	O – C_1	O – C_2	Cycle	J.D.	O – C_1	O – C_2
1	2415602	+ 8	+ 1	38	2418150	+ 1	+ 7
2	5674	+15	+ 4	39	8214	+ 4	+ 2
6	5936	+ 1	− 8	49	8896	+ 1	− 3
8	6106	+ 1	+24	55	9322	−19	+11
11	6285	+10	− 3	59	9573	− 3	−13
12	6350	+10	− 7			...	
16	6630	+14	− 1	65	2420016	−18	+18
17	6692	+11	− 8	82	1158	− 2	− 8
22	7043	+21	± 0	87	1504	+ 3	− 5
23	7122	− 5	+10	93	1862	+20	+ 9
28	7465	− 2	+ 9	97	2198	+15	+ 2
29	7532	± 0	+ 7				
33	7798	−10	− 1	113	3270	+ 1	−25
34	7858	−15	−10	119	3700	−15	− 8

The last two dates might be omitted from the discussion, since visual observations about that time indicate only slight and irregular variation and give no support to the supposed maximum on J.D. 2423700. There then remain in column 4 two large residuals of +24 and +18 days. The latter (cycle 65) has its counterpart −18 in column 3, so the observed date does not agree with either set of elements and is probably erroneous. The out-outstanding residual is that of +24 days at cycle 8, concerning which, in the absence of any information as to the observational material on which the date is based, no explanation can be usefully offered. It may be remembered, however, that when the total range of variation is sometimes as small as $0^{m}\cdot3$, a secondary inflection may not improbably be mistaken for a true maximum, so that too much importance need not be attached to the existence of one or two large discrepancies ; it is the good agreement of the great majority of dates which is of more importance. Taking the period 1901–1919

and ignoring the two large residuals mentioned above, the two largest discrepancies are 13 days and 11 days, and the mean divergence in column 4 is only $5^{d}\cdot8$, much too close an agreement to be the result of chance.

5. From the above considerations the following points may be emphasised: first, the visual observations of RU Aquarii made at Zaragoza in 1910–1911, showing a regular succession of maxima at intervals of about 69 days, render untenable the theory of an unsymmetrical light-curve, period 170 days, advanced in *Harvard Bulletin*, No. 848, at least if considered as a permanent characteristic of the star's variation; and secondly, the period deduced from the said visual observations when applied in a perfectly straightforward manner is found to satisfy the dates of maxima photographically determined at Harvard quite as well as (I think decidedly better than) the more complicated and seemingly less probable formula, involving an adjustment of phase in the middle of the period studied, as proposed in the aforementioned *Bulletin*.

This agreement holds good up to about 1919. Whether it represents a permanent feature of the variation is a matter which cannot yet be decided. There is evidence that at times the variation is of very small amplitude and apparently irregular, or the main periodicity may be masked by secondary terms or irregularities. It seems hardly probable that a star of such small range and short period (69^{d}–85^{d}) should exhibit a variation of a regular long-period type. Moreover the spectrum M*c*, combined with the above characteristics, would rather suggest a semi-regular variation of the class of *X* Herculis, characterised by a fairly definite periodicity which keeps recurring but alternates with periods of slight or irregular variation.

The visual maxima appear to take place somewhat earlier than the photographic, but the difference is only a few days. The colour-index is about $+1\cdot0$.

Zaragoza, Spain:
1931 *December.*

REMARKS ON PROFESSOR GERASIMOVIČ'S PAPER,* "LUMINOSITY CURVE OF EARLY B STARS."

Jan Schilt, Ph.D.

(Communicated by Professor Schlesinger.)

Applying a method devised by G. Strömberg (*Mount Wilson Contributions*, Nos. 395 and 396, 1931), we have the integral equation

$$F_1(x) = \frac{N_1}{N_2 N_3}\int_{-\infty}^{+\infty} F_2(x - M')F_3(M')dM',$$

giving a relation between the distribution functions F_1, F_2 and F_3 of the τ components of proper motions, linear velocities and absolute magnitudes

* *M.N.*, **91**, 537, 1931.

respectively, N_1, N_2 and N_3 being the total numbers of stars in the three distributions. We have:

$$x = \log \tau + 0{\cdot}2m, \quad y = \log V' - 1{\cdot}676, \quad x = M' + y, \quad M' = 0{\cdot}2M,$$

in which τ is the component of proper motion perpendicular to the direction of parallactic motion, m is the visual magnitude, V' is the linear velocity and M is the absolute magnitude. Without making an assumption as to the shape of the function involved, F_3 is solved numerically from the observed F_1 and F_2. It is assumed that a series of values of F_3 which gives by the use of the above equations a fair representation of F_1 will indeed represent the luminosity law. In applying this formula, however, extreme care must be used in ascertaining the extent to which the functions F_3 can be varied without violation of the observed function F_1. Gerasimovič finds for the F_3 curves multiple maxima, which may or may not be real, but the reality cannot be derived from his material. This contention will be proved in the present note. Let us adopt the computed distributions $F_1(x)$, as given by Gerasimovič. We then have:

$$F_1(x_1) = F_2(y_1)F_3(M_1'),$$
$$F_1(x_2) = F_2(y_2)F_3(M_1') + F_2(y_1)F_3(M_2'),$$
$$F_1(x_3) = F_2(y_3)F_3(M_1') + F_2(y_2)F_3(M_2') + F_2(y_1)F_3(M_3'), \text{ etc.}$$

If we change, for example, $F_3(M_l')$, $F_3(M_{l+1}')$, $F_3(M_{l+2}')$ by amounts A_l, A_{l+1}, A_{l+2} we obtain the following corrections to $F_1(x_n)$:

$$\Delta F_1(x_n) = (A_l + A_{l+1} + A_{l+2})F_2(y_{n-l}) + (A_{l+1} + 2A_{l+2})F_2'(y_{n-l}) + A_{l+2}F_2''(y_{n-l}),$$

where F_2' and F_2'' denote the first and second differences of F_2 respectively.

TABLE I

$F_3(M')dM'$

M	Bo and B2		B3		B4	
−4·1	0·0	0·0	0·0	0·0	0·0	0·0
−3·6	0·2	0·4	0·2	0·2	0·8	0·8
−3·1	33·3	32·2	6·1	6·1	1·4	1·4
−2·6	55·6	44·9	11·5	11·5	11·2	11·2
−2·1	29·4	42·1	20·3	20·3	41·8	33·8
−1·6	43·4	37·8	29·9	29·9	24·1	32·1
−1·1	13·0	19·0	34·4	34·4	28·9	28·9
−0·6	8·3	11·5	27·8	33·4	27·3	24·3
−0·1	10·2	8·4	35·6	29·6	2·2	20·4
+0·4	6·6	3·6	15·5	16·6	21·0	17·0
+0·9	0·0	0·0	13·6	12·6	15·7	13·4
+1·4	0·0	0·0	5·1	5·1	11·0	9·5
+1·9	0·0	0·0	0·0	0·0	12·8	5·4
+2·4	0·0	0·0	0·0	0·0	1·8	1·8
+2·9	0·0	0·0	0·0	0·0	0·0	0·0

The corrections to $F_1(x)$ thus depend on the differences of $F_2(y)$. (We can moreover choose A_l, A_{l+1} and A_{l+2} so that the coefficients of the first differences in $F_2(y)$ disappear.)

For the three cases treated by Gerasimovič I have made solutions based on functions F_3, obtained by smoothing out graphically the details of the curves F_3 as adopted by Gerasimovič.

The first figures in the columns of Table I are the values adopted by Gerasimovič. The second are obtained by smoothing graphically. The computed functions $F_1(x)$ and the observed ones are as follows:—

TABLE II

$F_1(x)dx$

Bo and B2			B3			B5		
O	G	S	O	G	S	O	G	S
2	1	1	2	0	0	2	0	0
2	4	3	4	1	1	3	1	1
5	5	5	5	3	2	6	4	3
5	8	8	5	5	5	6	6	6
9	11	11	9	8	8	12	10	10
9	13	14	9	12	12	12	14	14
14	18	17	14	18	17	19	19	19
19	22	22	19	23	23	25	25	25
39	35	34	33	30	30	25	32	31
55	53	52	47	37	37	21	40	40
70	64	63	31	47	46	42	49	49
35	69	70	39	57	57	55	58	58
120	73	73	94	67	67	68	70	69
66	75	74	91	78	77	85	77	79
84	75	76	87	87	84	99	91	89
127	84	81	79	89	89	115	84	88
46	82	80	98	90	89	55	81	84
57	68	70	53	85	82	48	77	76
49	60	61	87	72	71	48	60	66
67	53	50	80	59	62	51	57	54
28	41	41	43	46	46	79	47	43
17	35	33	31	34	34	28	35	32
30	23	22	13	22	26	40	28	23
24	14	13	6	14	14	18	20	15
7	7	7	14	8	8	3	10	10
4	5	4	6	5	5	13	8	6
0	0	2	2	3	3	11	4	3
0	0	1	1	2	2	3	2	2
0	0	0	4	1	1	6	1	1
2	0	0	0	1	1	0	1	0

The values under the headings O, G and S refer to the observed values and the computed ones by Gerasimovič and myself respectively. I have not tried to obtain a better representation of the observed values, but have only shown that the computed curves are practically the same whether we use the first or the second set of values for $F_3(M')$ as given in Table I.

Strömberg's method in deriving the luminosity law has the advantage of giving everything that can be obtained from the proper motions and radial velocities. If, however, it is applied without proper weighing of the observed data there is a danger of obtaining details which have no inherent significance.

Columbia University:
1931 *November.*

ON THE BEHAVIOUR OF THE BANDS OF *CH* AND *CN* IN THE SPECTRUM OF δ CEPHEI.

*P. Swings.**

(Communicated by Dr. Otto Struve.)

1. Quantitative spectrophotometric data are almost entirely lacking in the case of the Cepheid variables, in spite of the fact that such information would be invaluable for a discussion of the pulsation hypothesis and of the conditions of temperature, pressure and ionisation as functions of phase in the light-curve. Although several spectrophotometric investigations have appeared within the last few years, most of these are of a more or less qualitative character, and some of them are based upon spectrograms taken with very small dispersion and with objective prisms. Lack of resolving power is especially dangerous in the case of molecular bands. In a recent note, P. Swings and O. Struve † have shown that it is necessary carefully to avoid blends with atomic lines if one wishes to study the bands of *CH* and *CN* in the spectra of stars of types F and G. In the same note the conclusion was reached that, contrary to the general idea, the bands of *CH* and *CN* disappear at very nearly the same place in the spectral sequence, viz. spectral class F8. The astronomical literature contains numerous errors which have been caused by blends of molecular and atomic lines; for example, the *CH* band was believed by some investigators to persist as far as Fo or even earlier. We shall enumerate some of these errors in the last section of this note.

On the other hand, many investigations have been made with the use of non-standardised plates, in which case the microphotometric deflections can give qualitative results only, and even these may be vitiated if the measured photographic blackenings do not fall in a good region of the Hurter and Driffield curve of the emulsion.

The best method of obtaining definite and useful results for the spectra of Cepheids consists in using a slit spectrograph of sufficient dispersion and in taking spectra on well-standardised plates of fine grain.

2. We have made a careful investigation of the bands of *CH* and *CN* in the spectrum of 27 δ Cephei. We used slow Eastman process plates in connection with the Bruce spectrograph of the Yerkes Observatory, the

* Advanced Fellow of the C.R.B. Educational Foundation.

† P. Swings and O. Struve, *Physical Review*, 1932 January.

dispersion being 25 A. per mm. On each plate we impressed a photometric scale of intensity marks. For the examination of the bands we used the regions indicated in the above-mentioned paper of P. Swings and O. Struve. The spectrograms were analysed with the registering microphotometer of the Yerkes Observatory, according to the method adopted by C. T. Elvey and O. Struve.

Table I

Plate	Date U.T. 1931 d h m	Phase	Observer	Sp. Type
1 R 9931	Oct. 22 0 12	0·09	*σ*	F6–F7
9933	22 6 00	0·13	*σ*, *S*	F7
9941	Nov. 2 3 48	0·16	*σ*, *O*, *S*	F7
9910	Oct. 18 0 54	0·35	*σ*, *Sw*	G3
9912	18 5 15	0·38	*σ*, *Sw*, *S*	G3
9917	19 1 08	0·53	*M*, *S*	G5
9945	Nov. 4 5 34	0·55	*Sw*, *S*	G5
9919	Oct. 19 7 01	0·58	*σ*, *M*, *S*	G5
9903	9 5 21	0·71	*σ*, *M*, *S*	G6
9954	Nov. 16 1 40	0·75	*σ*, *Sw*, *S*	G6
9955	16 3 17	0·76	*σ*, *Sw*, *S*	G6
9921	Oct. 21 0 14	0·90	*M*, *Sw*	F8–F9
9950	Nov. 6 2 51	0·90	*M*, *S*	F8–F9
9909	Oct. 10 4 27	0·90	*Sw*, *S*	F8–F9
9922	21 1 40	0·91	*M*, *Sw*, *S*	F8
9923	21 2 53	0·92	*Sw*, *S*	F8
9924	21 3 41	0·92	*Sw*, *S*	F8
9925	21 4 28	0·93	*Sw*, *S*	F7–F8
9926	21 5 24	0·94	*Sw*, *S*	F7–F8
9927	21 6 42	0·95	*σ*, *Sw*, *S*	F7
9928	21 8 10	0·96	*σ*, *S*	F7
9929	21 9 41	0·97	*σ*, *S*	F6
9937	Nov. 1 2 54	0·97	*σ*, *O*, *S*	F6
9938	1 4 48	0·98	*σ*, *O*, *S*	F4–F5

The names of the observers are indicated as follows : *σ* = O. Struve ; *O* = K. Ogrodnikoff ; *M* = W. W. Morgan ; *Sw* = P. Swings ; *S* = F. R. Sullivan.

Table I contains a list of the plates examined, giving the serial number in the Yerkes collection, the date U.T., and the spectral type as deduced from the diagram of variation of spectral type of δ Cephei by H. Shapley * after a correction of 0·4 spectral class (following Miss Payne †) had been applied.

3. In fig. 1 the full-drawn line shows the light-curve of δ Cephei as given by M. Luizet.‡ We have indicated on this curve the points corresponding to maximum, minimum and mean volumes by using the curve of radial velocities and also the pulsation diagram given by W. Baade. §

The dotted line shows the light-curve as it would be if the star retained

* H. Shapley, *Astrophysical Journal*, **44**, 290, 1916.

† C. H. Payne, *The Stars of High Luminosity*, p. 309.

‡ M. Luizet, *Annales de l'Université de Lyon*, Nouvelle série, fasc. 33, 1912.

§ W. Baade, *Astronomische Nachrichten*, **228**, 5468, p. 359, 1926.

always the same (mean) volume ; this curve must cross the full line in the points corresponding to mean volume. The corrections (without considering the sign) reach maximum when the star is of maximum or minimum volume ; the actual values of these corrections were determined by means of the formula given by P. ten Bruggencate * ; they are, of course, only approximate.

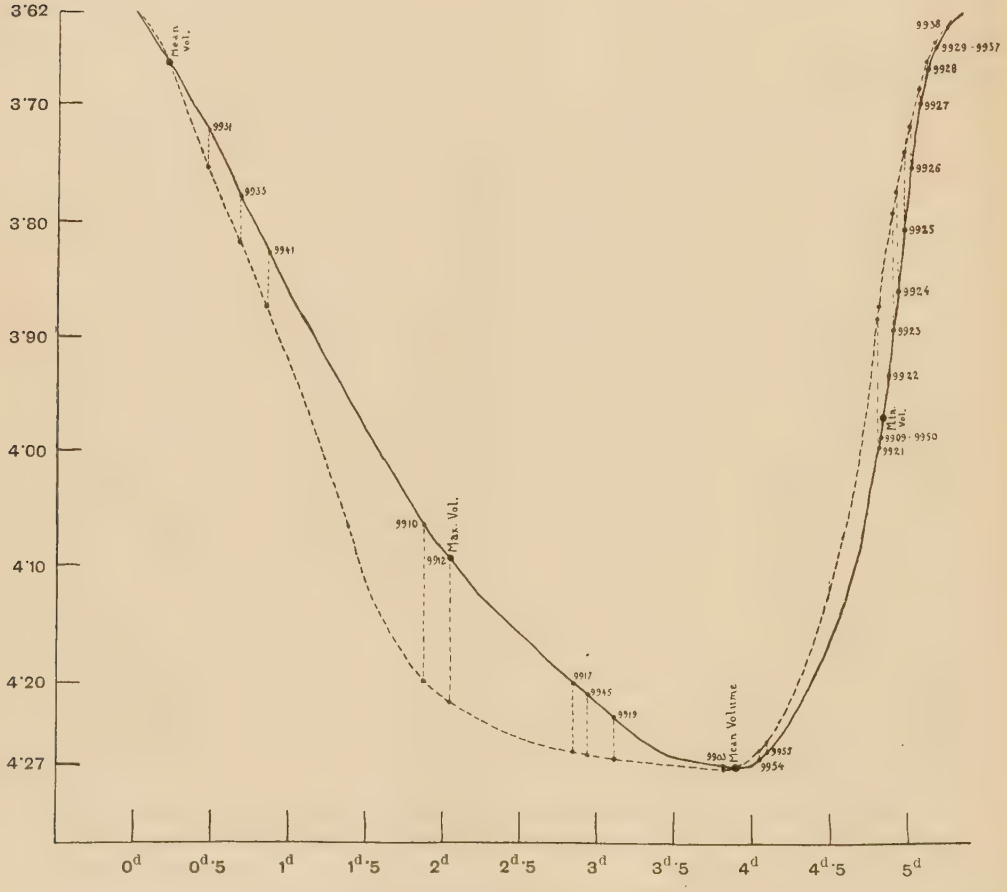


Fig. 1.—*Light-curve of δ Cephei*

4. Examination of the plates revealed immediately that the *CH* and *CN* bands undergo quite similar variations. Both are absent near maximum light ; they appear at about phase 0·16, or $0^d·85$ after the maximum of light (plate 9941) ; their intensities, extremely faint on plate 9941, are gradually increasing and reach their maximum near phases 0·70–0·75 (plates 9903, 9954, 9955 ; light minimum). The intensities thereafter decrease, and the bands disappear before phase 0·90.

It is of especial interest to note that plate 9941, on the descending branch of the curve, shows a very faint *CH* absorption band, while plates 9924 and 9925, taken on the ascending branch when the star was of the same apparent

* P. ten Bruggencate, *Die veränderlichen Sterne*, pp. 34, 37.

magnitude, show no trace of *CH*. This seems to indicate that two states of the same apparent brightness do not necessarily have equal *CH* and *CN* absorptions ; it agrees with the pulsation hypothesis, according to which each point on the descending curve would correspond to a lower temperature than the corresponding point of the same brightness on the ascending curve.

5. On four of the best plates we have measured the absorption of the *CH* molecules at λ 4311A (this being a region free from blends, as shown in the paper by P. Swings and O. Struve). The measured values are given in Table II.

TABLE II

Plate	Phase	Absorption Per cent.	N*	Sp. Class †	Temp.‡
1 R 9912	0·38	12·5	39	G3	5100
9945	0·547	17	72	G5	4700
9903	0·71	20	100	G6	4600
9954	0·754	20	100	G6	4600

For comparison we have also measured the *CH* absorption for the same wave-length in the solar spectrum ; we found 41 per cent. This indicates an " absolute magnitude effect " for the molecular bands, δ Cephei and the Sun being respectively a super-giant and a dwarf.

6. Knowing these absorptions, the question arises of determining for the different phases the ratios of the numbers of molecules of *CH* in the stellar atmosphere.§

Let $N_{j''}$ be the number of *CH* molecules per cubic centimetre, which are in the energy state j'' (j'' thus represents the ensemble of the electronic, vibrational and rotational energies), and let $B_{j''j'}$ be the probability of transition of the molecule from the j'' level to a j' level of higher energy. If λ_0 is the wave-length of the centre of the $j''j'$ absorption line, the absorption coefficient σ is a function ϕ of the difference $\lambda - \lambda_0$ between the centre of the line and the examined point ; σ is also proportional to the product $N_{j''}B_{j''j'}$. Hence

$$\sigma(\lambda) = C\phi(\lambda - \lambda_0)N_{j''}B_{j''j'},$$

C being a constant depending upon λ_0 and the molecular properties. The residual intensity for each wave-length λ will be expressed by

$$I(\lambda) = I^*(\lambda)\,\psi(\sigma),$$

$I^*(\lambda)$ being the intensity for wave-length λ corresponding to the continuous spectrum and $\psi(\sigma)$ a function of σ. This function differs according to

* See end of § 6.

† H. Shapley, *loc. cit.*

‡ Russell, Dugan and Stewart, *Astronomy*, **2**, p. 753. The temperatures are expressed in absolute degrees.

§ This procedure is similar to that followed by O. Struve and C. T. Elvey for the comparison of the relative intensities of the components of a Si III triplet in stellar and laboratory spectra, *Astrophysical Journal*, **72**, 267, 1930.

whether we consider the case of pure absorption without any re-emission of the same wave-length or the case of a diffusing atmosphere. But, as was shown by Unsöld,* the difference between these two ψ functions is not great. We shall adopt here the formula

$$I(\lambda) = I^*(\lambda)\, e^{-\int \sigma dl},$$

l being the thickness of the layer ; this formula may be written

$$I(\lambda) = I^*(\lambda)\, e^{-\sigma l}$$

if σ does not depend upon the position in the reversing layer. Unsöld † has shown also that the expression for σ in the case of pure radiation damping is

$$\sigma = K \frac{N_{j''} B_{j''j'}}{(\lambda - \lambda_0)^2},$$

K being a constant.

The whole energy absorbed in the line is thus

$$E = I_0 \int_{-\infty}^{+\infty} \left\{ 1 - e^{-\frac{K N_{j''} B_{j''j'} l}{(\lambda - \lambda_0)^2}} \right\} d\lambda = K_{j''j'} \sqrt{N_{j''} B_{j''j'} l},$$

I_0 being the intensity of the incident radiation, and $K_{j''j'}$ another constant.

We must pay attention to the fact that $N_{j''}$ is a certain proportion of the total number N of molecules CH in the atmosphere ; we shall write

$$N_{j''} = \alpha_{j''} N.$$

Let us consider two different temperatures included between our limiting temperatures for the determinations of absorption, *i.e.* between 4600° and 5100° C. The ratio $\frac{E_1}{E_0}$ of the absorbed energies is

$$\frac{E_1}{E_2} = \sqrt{\frac{a_{1j''}}{a_{2j''}}} \sqrt{\frac{B_{1j''j'}}{B_{2j''j'}}} \sqrt{\frac{l_1}{l_2}} \sqrt{\frac{N_1}{N_2}}. \qquad (1)$$

We shall examine the importance of each of these factors.

(*a*) $\sqrt{\frac{a_{1j''}}{a_{2j''}}}$. An increase of temperature changes the value of the rotational quantum number m_0'' of the greatest number of molecules ; this change gives rise to a displacement of the maximum of intensity in the rotational lines of a partial band.

The Boltzmann formula gives

$$\mathcal{J} \frac{w_m^2}{2} = \tfrac{1}{2} kT,$$

where $\mathcal{J}$ is the moment of inertia of the molecule (with respect to the axis of

* A. Unsöld, *Zeitschrift für Physik*, 59, 363, 1930 ; see also O. Struve and C. T. Elvey, *loc. cit.*

† *Ibid.*, 44, 793, 1927 ; 46, 768, 1928. See also R. Ladenburg, *ibid.*, 4, 454, 1921.

rotation), w_m the frequency of rotation and k the Boltzmann constant. As a first approximation we shall use the old quantum theory, which gives us

$$\mathcal{J} w_m = \frac{m_0'' h}{2\pi}.$$

Thus we get

$$m_0'' = \frac{2\pi}{h}\sqrt{kT\mathcal{J}}.$$

For two different temperatures, T_1 and T_2, we find

$$\frac{m_0''(T_1)}{m_0''(T_2)} = \sqrt{\frac{T_1}{T_2}}.$$

In our problem, the maximum value of this ratio is

$$\frac{m_0''(5100)}{m_0''(4600)} = 1{\cdot}05.$$

The influence of this factor is not worth considering, especially if the rotational lines which we consider are not too far from the band-head.

(*b*) $\sqrt{\frac{B_{1j''j'}}{B_{2j''j'}}}$. This ratio is almost certainly very near unity.

(*c*) $\sqrt{\frac{l_1}{l_2}}$. It is almost certain that the thickness of the absorbing layer does not change appreciably.*

(*d*) $\sqrt{\frac{N_1}{N_2}}$. This is certainly the most important factor in equation (1). We shall therefore assume that the intensity of absorption is proportional to $\sqrt{N}$.

Our reasoning implies that there is only one absorption line at the point of the spectrum where the absorption is measured. In reality this point is a blend of several *CH* lines ; but this does not change the result. The absorbed energy is

$$E = \Sigma K_{j''j'} \sqrt{N_{j''} B_{j''j} l} = \sqrt{N}(\Sigma K_{j''j'} \sqrt{a_{j''} B_{j''j'} l}),$$

the Σ being applied to the different $(j''j')$ lines ; $K_{j''j'}$ has very nearly the same value for all the blended lines. On the other hand, we have shown above that l, $a_{j''}$ and $B_{j''j'}$ do not change much when T varies between 5100 and 4600. Thus we may still consider E proportional to $\sqrt{N}$.

Accordingly we find for the relative numbers of molecules the values given in column 4 of Table II.

7. We shall now show that these values of the numbers of molecules

* P. ten Bruggencate, *loc. cit.*, p. 34.

are in close agreement with the values computed by means of the equation of dissociation-equilibrium of *CH*. This equation is *

$$\log K_p = -\frac{110{,}000}{4{\cdot}541T} + 1{\cdot}5 \log T + \log\left(1 - e^{-\frac{4000}{T}}\right) - 0{\cdot}40,$$

K_p being the coefficient of equilibrium $= \frac{p_C p_H}{p_{CH}}$, where p_C and p_H are the partial pressures of *C* and *H* (assumed to be constant) ; *T* is the absolute temperature.

By applying this equation to various temperatures we find :

TABLE III

Sp. Class	*T*	log K_p	*N*
G6	4600	9·6267	100
G5	4700	9·7461	76
	4800	9·8606	59
	4900	9·9708	45
G4	5000	0·0764	36
	5100	0·1781	28
	5200	0·2758	23
	5300	0·3700	18
	5400	0·4609	15

The measured numbers of molecules are 100, 72, 39, corresponding to the following temperatures : 4600°, 4720° and 4950° C., *i.e.* to spectral types G6, G5 and G3.

The agreement between theory and observation seems perfect.

8. We shall close by pointing out a few uncertain conclusions found in the astronomical literature concerning the *CH* and *CN* bands in Cepheids.

(*a*) In a recent paper † on three Cepheid variables (η Aquilæ, RT Aurigæ and α Ursæ Minoris), Dr. A. V. Douglas points out that the variation of the *CN* band is in phase with the periodic variation of many lines of ionised elements and of the Balmer lines, and that there is a great difference of phase between the variation of the *CH* and *CN* bands. She suggests, in order to explain this phenomenon, that the molecule *CN* is in the ionised state.

From the identical behaviour of the *CH* and *CN* bands in δ Cephei it seems extremely probable that this is not the case. Dr Douglas's plates were not standardised, so that her quantitative determinations are rather doubtful ; furthermore, it is certain that not enough caution was used to eliminate blends.

(*b*) For the same reason we feel little confidence in the values of the intensity of the *CH* band and in the curve of variation of the ratio $\frac{\text{int. of G band}}{\text{int. of } Cr\ 4289}$ given by F. Henroteau and A. V. Douglas.‡

* P. Swings and O. Struve, *loc. cit.* † A. V. Douglas, *M.N.*, **90**, 798, 1930.

‡ F. Henroteau and A. V. Douglas, *Publications of the Dominion Observatory, Ottawa*, **9**, No. 7, Table III, and fig. 9, 1929.

(*c*) It is also almost certain that the results obtained by G. Tiercy * for SU Cassiopeiæ are vitiated by blends in the case of the G band. † On account of the small dispersion used by him, it is certain that what he calls the " G band " is mostly made up of blends of atomic lines. This explains his observation that in a spectrum of class F5 the G band is more conspicuous than in the preceding spectra of classes F1–F2 and F1. We have shown formerly ‡ that the G band disappears at F8.

(*d*) Recent papers of P. ten Bruggencate § on the variations of the spectra of several Cepheids also include a discussion of the behaviour of the *CH* band. Some of his results are probably caused by the fact that his microphotometric deflections are not essentially due to the *CH* molecules.

(*e*) Miss C. H. Payne || points out an increase in intensity of the *CH* band with decrease of pressure at constant temperature. This seems difficult to explain. It may perhaps be due to blends with atomic lines. Unfortunately our observations do not settle this question.

It is a great pleasure to record my gratitude to Professor O. Struve for helpful advice and criticism at all times.

Yerkes Observatory :
1931 *November* 25.

NOTES ON THE CONSTITUTION OF THE STARS : ADDENDUM.

H. N. Russell.

I am greatly indebted to Professor H. Vogt for calling my attention to the fact that he published in 1926 (*A.N.*, **226,** 301) the proposition that a series of stars of fixed composition must exhibit exact relations connecting mass, luminosity, etc. ; and certain other general theorems. Had I known of this, I would have eliminated in my recent paper (*M.N.*, **91,** 951, 1931) all reference to my publication of these propositions a year later in a text-book. The humorous form of this reference expressed my conviction that Professor Vogt was quite unaware of my publication, and that, in the circumstances, he was under no obligation to know of it.

* G. Tiercy, *Publicazioni Oss. Arcetri*, fasc. 44, 7, 1927.

† This is, of course, only a minor detail in Tiercy's papers, which contain numerous results of very great interest.

‡ P. Swings and O. Struve, *loc. cit.*

§ P. ten Bruggencate, *Annalen v. d. Bosscha Sterrenwacht, Lembang*, **5,** 1st part, 1931 ; *Die veränderlichen Sterne, Ergebnisse der exakten Naturwissenschaften*, **10,** 33–44, 1931.

|| C. H. Payne, *The Stars of High Luminosity*, p. 214.